Infrastructure Commission (INFCOM)

Standing Committee on Measurements, Instrumentation and Traceability (SC-MINT)

Expert Team on Quality, Traceability and Calibration (ET-QTC)

Calibration of Pressure Instruments

Part-4: Calculation of Uncertainty Budget (1)

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Content

Part 1:

- Introduction to this topic and historical information
- Concepts and definitions

Part 2:

- Methods of measurement
- Pressure standards in calibration laboratory

Part 3:

- Comprehensive calibration procedure
- Calibration equipment and data acquisition

Part 4:

- **Measurement uncertainty contributions part #1**

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Measurement uncertainty contributions

- there are numerous influences (variables) that contribute to uncertainty

Standard Barometer / Reference (SB)	Device under test / Barometer under test (DUT)	Process	Staff
calibration certificate		stability of the controller	education
long-term stability		correlation	training
environmental conditions	environmental conditions	environmental conditions	experience
hysteresis	hysteresis		mental condition
repeatability	repeatability	altitude difference	physical condition
resolution	resolution	temperature difference	environment
interpolation		uncertainty model	care
sampling rate	sampling rate	computation	
data acquisition	data acquisition	tidiness	
		cleanliness	

- and for sure there will be some more influences

Measurement uncertainty contributions

- temporal inhomogeneity of pressure generation
  - the stability of the pressure generation and control shall be taken into account in the uncertainty analysis
  - due to pressure generation and control, some correlation exists between the measured values of the SB and the DUT
    - this correlation means their measurements are somehow influenced in the same way at the same time

Measurement uncertainty contributions

- temporal inhomogeneity of pressure generation

  - there are several ways to address this problem
    - one can use the data of each calibration and process them further
    - or one can carry out regular characterization tests of the pressure controller

- *to simplify here, we assume an ideally stable pressure controller*
- *in an additional document, it is explained how to evaluate the stability of pressure generation and control and how correlated pressure values should be handled with unstable pressure controllers*

Measurement uncertainty contributions

- the input variables x_i for the example model are shown in the formula

$$\Delta p = p_{DUT} - p_{SB} + \delta_{tempDUT} + \delta_{altDUT} + \delta_{hystDUT} + \delta_{repeatDUT} + \delta_{resolDUT} - \delta_{tempSB} - \delta_{calSB} - \delta_{driftSB} - \delta_{hystSB} - \delta_{repeatSB} - \delta_{resolSB} - \delta_{interpolSB} - \delta_{Pstab}$$

- the best estimate is always given; so p_{DUT} and p_{SB} are mean values and in the best case all other input variables have a value, but mostly zero
  - they can have a real numerical value, e.g. if corrections have to be made
    - e.g. δ_{calSB} is a correction taken from the calibration certificate

Measurement uncertainty contributions

- each input variable has a measurement uncertainty $u(x_i)$ associated with it

$$u_c(y) = \sqrt{\sum_{i=1}^{N} (c_i \cdot u(x_i))^2}$$

$$u_c(y) = \sqrt{\begin{array}{c} u(\delta p_{DUT})^2 + u(\delta_{tempDUT})^2 + u(\delta_{altDUT})^2 + u(\delta_{hystDUT})^2 + u(\delta_{repeatDUT})^2 + u(\delta_{resolDUT})^2 + \\ u(\delta p_{SB})^2 + u(\delta_{tempSB})^2 + \\ u(\delta_{calSB})^2 + u(\delta_{driftSB})^2 + u(\delta_{hystSB})^2 + u(\delta_{repeatSB})^2 + u(\delta_{resolSB})^2 + u(\delta_{interpolSB})^2 + u(\delta_{Pstab})^2 \end{array}}$$

$$U = u_c(y) \cdot k_p$$

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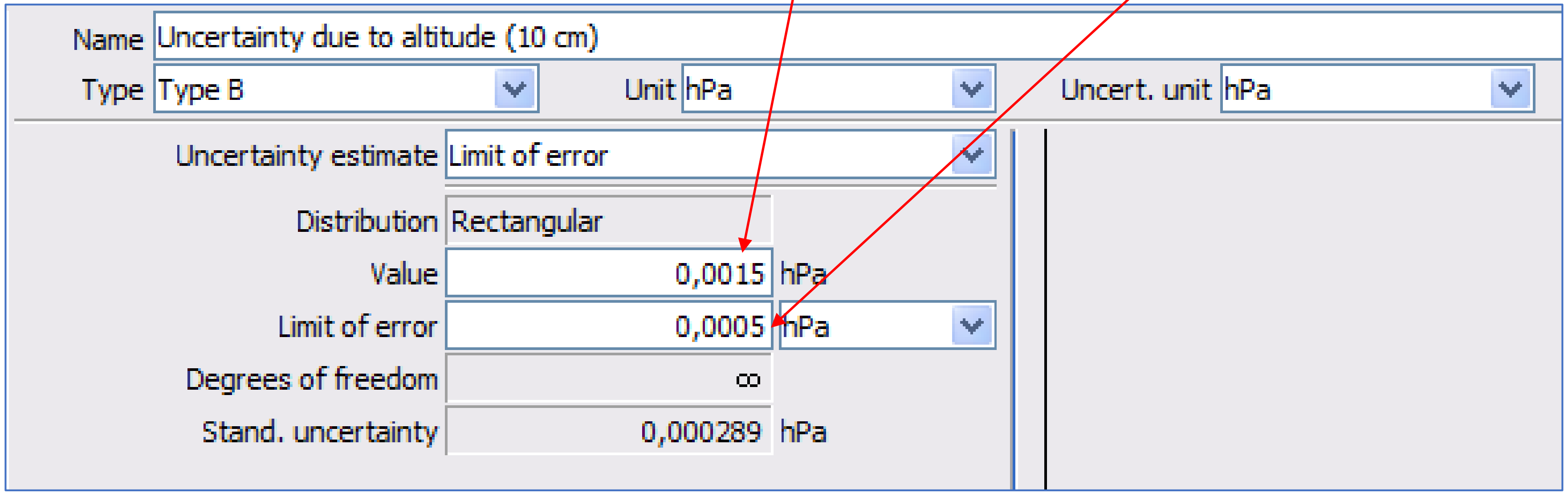
Measurement uncertainty contributions

$$\Delta p = \cdots + \delta_{altDUT} + \cdots$$

$$u(\delta_{altDUT})$$



Measurement uncertainty contributions

- consideration of the input variables

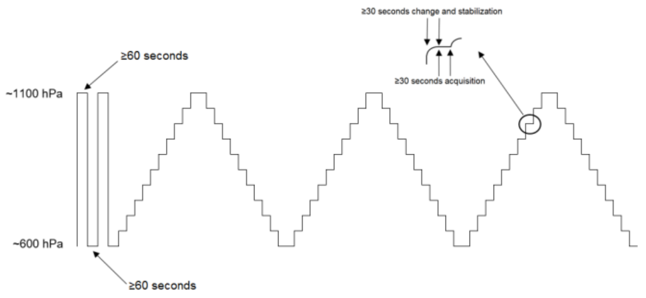
  - Type A:

    - the values x_i and $u(x_i)$ are determined on the basis of repeated measurements and their statistical evaluation

    - for each calibration point of the six measurement series, the mean value and the standard uncertainty are calculated from the 60 individual values

    - we thus obtain 66 mean values and standard uncertainties for the standard barometer and for device under test

Measurement uncertainty contributions



- consideration of the input variables
    - Type A:
        - from the six mean values of the same level of the six measurement series, we calculate a mean value which then ultimately includes all the measured values

p_{DUT} and p_{SB}

        - the standard uncertainties are not averaged, but the maximum value of each of the six standard uncertainties is used

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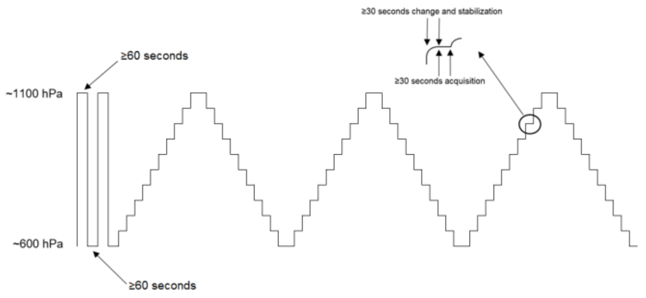
Measurement uncertainty contributions



- consideration of the input variables
  - Type A:
    - best estimate is the arithmetic mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$u(\bar{x}) = \frac{s(x_i)}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)} \cdot \sum_{i=1}^{n} (x_i - x)^2}$$

Measurement uncertainty contributions

- Type A: each mean value is based on sixty individual values

mean values Standard Barometer hPa						
M1_up	M2_down	M3_up	M4_down	M5_up	M6_down	mean
602,648	602,647	602,645	602,645	602,644	602,643	602,645
653,093	653,090	653,091	653,090	653,090	653,087	653,090
703,674	703,673	703,673	703,672	703,671	703,669	703,672
754,563	754,563	754,562	754,560	754,559	754,558	754,561
794,920	794,919	794,919	794,917	794,916	794,914	794,918
845,501	845,501	845,500	845,499	845,497	845,495	845,499
895,473	895,473	895,472	895,471	895,468	895,467	895,471
945,848	945,848	945,846	945,846	945,843	945,843	945,846
996,357	996,356	996,355	996,354	996,351	996,350	996,354
1045,531	1045,530	1045,529	1045,528	1045,524	1045,524	1045,528
1095,975	1095,974	1095,973	1095,972	1095,969	1095,969	1095,972

mean values device unter test hPa						
M1_up	M2_down	M3_up	M4_down	M5_up	M6_down	mean
602,700	602,694	602,696	602,687	602,684	602,682	602,690
653,138	653,125	653,123	653,140	653,130	653,130	653,131
703,713	703,711	703,703	703,709	703,712	703,712	703,710
754,605	754,601	754,599	754,592	754,592	754,602	754,598
794,966	794,960	794,960	794,961	794,955	794,957	794,960
845,541	845,544	845,538	845,538	845,534	845,538	845,539
895,507	895,512	895,510	895,512	895,498	895,504	895,507
945,883	945,882	945,878	945,877	945,876	945,878	945,879
996,387	996,391	996,385	996,385	996,382	996,382	996,385
1045,557	1045,554	1045,548	1045,551	1045,547	1045,551	1045,552
1096,005	1096,001	1095,996	1095,995	1095,989	1095,994	1095,997

$$\Delta p = p_{DUT} - p_{SB}$$

Measurement uncertainty contributions

- Type A: each mean value is based on sixty individual values

standard uncertainties Standard Barometer hPa						
M1_up	M2_down	M3_up	M4_down	M5_up	M6_down	max
5,56E-05	2,24E-05	0,00E+00	4,47E-05	1,08E-11	3,18E-05	5,56E-05
3,33E-05	0,00E+00	4,86E-05	1,11E-11	5,63E-05	5,59E-05	5,63E-05
1,12E-11	5,42E-05	0,00E+00	5,59E-05	4,47E-05	1,10E-11	5,59E-05
4,17E-05	5,52E-05	3,35E-05	3,31E-05	3,40E-05	5,66E-05	5,66E-05
4,36E-05	4,34E-05	5,56E-05	1,11E-11	4,47E-05	3,31E-05	5,56E-05
3,35E-05	5,27E-05	0,00E+00	5,59E-05	4,50E-05	4,44E-05	5,59E-05
2,04E-05	5,52E-05	4,47E-05	3,33E-05	3,27E-05	4,24E-05	5,52E-05
1,10E-11	5,52E-05	1,12E-11	5,52E-05	1,04E-05	4,36E-05	5,52E-05
2,16E-05	5,59E-05	2,21E-05	3,31E-05	5,49E-05	5,59E-05	5,59E-05
3,35E-05	5,49E-05	4,47E-05	1,10E-11	1,04E-05	5,33E-05	5,49E-05
5,56E-05	0,00E+00	2,24E-05	1,12E-11	5,63E-05	0,00E+00	5,63E-05

standard uncertainties device under test hPa						
M1_up	M2_down	M3_up	M4_down	M5_up	M6_down	max
2,76E-03	3,12E-03	3,13E-03	3,00E-03	2,68E-03	2,98E-03	3,13E-03
2,84E-03	2,38E-03	3,01E-03	2,89E-03	2,64E-03	2,21E-03	3,01E-03
2,55E-03	2,76E-03	2,55E-03	2,53E-03	2,32E-03	2,61E-03	2,76E-03
1,92E-03	2,66E-03	2,58E-03	2,42E-03	2,79E-03	2,35E-03	2,79E-03
2,42E-03	2,25E-03	2,33E-03	2,28E-03	2,24E-03	2,08E-03	2,42E-03
2,45E-03	2,25E-03	2,24E-03	1,86E-03	2,08E-03	2,40E-03	2,45E-03
2,12E-03	2,01E-03	1,84E-03	1,96E-03	1,91E-03	2,02E-03	2,12E-03
1,79E-03	2,12E-03	1,64E-03	1,84E-03	1,83E-03	1,80E-03	2,12E-03
1,63E-03	1,68E-03	1,54E-03	1,65E-03	1,34E-03	1,69E-03	1,69E-03
1,77E-03	1,63E-03	1,57E-03	1,42E-03	1,56E-03	1,58E-03	1,77E-03
1,21E-03	1,54E-03	1,19E-03	1,51E-03	1,52E-03	1,45E-03	1,54E-03

$$u(\bar{q}) = \frac{s(q_k)}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)} \cdot \sum_{k=1}^{n} (q_k - \bar{q})^2}$$

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Measurement uncertainty contributions

- consideration of the input variables
  - Type B:
    - the values x_i and $u(x_i)$ are determined on the basis of other sources and their processing (manufacturer's data; limit values; parameters known from previous investigations; literature values)

$$u(\delta_{tempDUT}), u(\delta_{altDUT}), u(\delta_{hystDUT}), u(\delta_{repeatDUT}), u(\delta_{resolDUT}),$$

$$u(\delta_{tempSB}), u(\delta_{calSB}), u(\delta_{driftSB}), u(\delta_{hystSB}), u(\delta_{repeatSB}), u(\delta_{resolSB}), u(\delta_{interpolSB}), u(\delta_{Pstab})$$

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Measurement uncertainty contributions

- Type B:

Source (related to the standard)	Name of uncertainty	type	distribution	Description
environmental conditions	$u(\delta_{tempSB})$	B	rectangular	• influence of the ambient temperature on the SB if not the same temperature as the one used for the calibration
calibration certificate	$u(\delta_{calSB})$	A	normal	• the calibration laboratory has determined a pressure-dependent function for the uncertainty • here, however, the highest value for the entire range is used (conservative)
long-term stability	$u(\delta_{driftSB})$	B	rectangular	• the maximum drift is the highest value of the drifts found between two calibrations, either over the SB life, or alternatively over the more recent calibration intervals (which may be interesting if the SB has stabilised with age)
hysteresis	$u(\delta_{hystSB})$	B	rectangular	• the hysteresis was taken from the last calibration certificate. • Rectangle if SB is a piston pressure gauge.
repeatability	$u(\delta_{repeatSB})$	B	rectangular	• the repeatability was taken from the last calibration certificate
resolution	$u(\delta_{resolSB})$	B	rectangular	• the display has two decimal places; the last number can be rounded up or down
interpolation	$u(\delta_{interpolSB})$	B	rectangular	• the SB was calibrated at given points but it is used at any point over its calibration range • the uncertainty due to the linear interpolation of the corrections between 2 calibration points of the SB

Measurement uncertainty contributions

- Type B:

source	Name of uncertainty	type	distribution	description
Stability of the pressure control	$u(\delta_{Pstab})$	B	rectangular	• **Reminder:** *In this concrete example we assume a very stable pressure controller, so that the value should not greatly influence the measurement uncertainty See other document for further explanation.*
environmental conditions	$u(\delta_{tempDUT})$	B	rectangular	• temperature differences within the pressure system can cause pressure differences on the calibration object
altitude difference	$u(\delta_{altDUT})$	B	rectangular	• SB and DUT are installed at the exact same altitude. But a possible small height difference between the pressure cell inside the SB and that inside the DUT induces a pressure difference
resolution	$u(\delta_{resolDUT})$	B	rectangular	• DUT with digital output: the last significant digit from the digital output can be rounded up or down
hysteresis	$u(\delta_{hystDUT})$	B	rectangular	• the procedure proposed in Euramet CG-17 is used here (formulas 22a, 23a, 24)
repeatability	$u(\delta_{repeatDUT})$	B	rectangular	• the procedure proposed in Euramet CG-17 is used here (formula 28)

Measurement uncertainty contributions

- consideration of the input variables

  - Type B: $u(\delta_{tempDUT})$
    - in general, it is recommended to keep the temperature stable at ±1 K throughout the calibration process
    - temperature differences in the area between the SB and DUT can cause density and thus pressure differences in the pressure tube
      - roughly estimated, a 1 K temperature difference leads to a pressure difference 0.05 Pa and a corresponding uncertainty of 0.03 Pa, which can be deemed negligible

Measurement uncertainty contributions

- Type B: $u(\delta_{tempSB})$
  - the SB has been calibrated at a given temperature $Tcal$, and it is used as a standard in the laboratory at any temperature within $T0\ \pm\ \Delta°K$, as fixed for operational conditions of the laboratory
  - as the measurement of the SB is not perfectly internally compensated for the influence of temperature, this influence shall be evaluated and taken into account

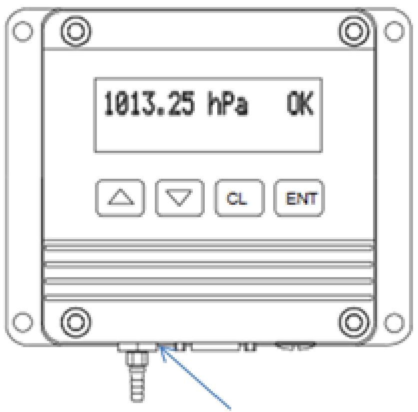
$$u(\delta_{tempSB}) = \frac{\Delta p_{temp}}{\sqrt{3}}$$

Measurement uncertainty contributions

- Type B: $u(\delta_{calSB})$
  - is an uncertainty taken from the calibration certificate

$$u(x_i) = \frac{u(\delta_{calSB})}{2}$$

Measurement uncertainty contributions



- Type B: $u(\delta_{altDUT})$
  - nevertheless, an uncertainty $u(\delta_{altDUT})$ of even a few mm should be taken into account for the height alignment
  - pressure difference due to height difference is estimated with: $\delta_{altDUT} = \rho \ * g \ * \delta h$ where ρ is air mass density $(\sim 1.2 kgm^{-3})$, g is gravity $(\sim 9{,}81 ms^{-2})$ and δh is the height difference in (m)

$$u(\delta_{altDUT}) = \frac{\Delta p_{altitude}}{\sqrt{3}}$$

  - 4 mm will result in an uncertainty of 0.05 Pa

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Measurement uncertainty contributions

- Type B: $u\left(\delta_{driftSB}\right)$

  - almost all instruments and measuring devices, and therefore standard barometers too, are subject to drift
  - uncertainty due to the drift shall always be taken into account
  - the drift can be estimated using the history of calibration certificates, comparing the results of subsequent regular calibrations
  - for each pressure level p_i the difference $diffcor_{i,n}$ is calculated between the correction found at a calibration CAL_n and the one found at the previous calibration CAL_{n-1}

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Measurement uncertainty contributions

- Type B: $u(\delta_{driftSB})$

  - if D_n is the drift between the calibrations CAL_n and CAL_{n-1}, then: $D_n = max(diffcorr_n)$
  - the drift D_n is calculated for all couples of calibration CAL_n and previous calibration CAL_{n-1} over the life of the SB
  - the maximum of all D_n is taken (possibly discarding the first year(s) if the SB has improved after a less stable youth)

$$u(\delta_{driftSB}) = \frac{max(D_n)}{\sqrt{3}}$$

Calibration of Pressure Instruments
End of Part 4

Thank you.

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