Infrastructure Commission (INFCOM)

Standing Committee on Measurements, Instrumentation and Traceability (SC-MINT)

Expert Team on Quality, Traceability and Calibration (ET-QTC)

Calibration of Temperature Instruments

Part-4: Uncertainties in Calibration by Comparison

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Definitions: Unit of temperature

The Unit of Temperature in the International System of Units is the **kelvin** (K)

The current definition of the kelvin was agreed by the 26th CGPM (November 2018) and came into force the 20th May 2019 [(1)]:

The kelvin, symbol K, is the SI unit of thermodynamic temperature (*T*). It is defined by taking the fixed numerical value of the Boltzmann constant *k* to be 1.380 649 x 10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} K^{-1}, where the kilogram, metre and second are defined in terms of *h*, *c* and $\Delta\nu_{Cs}$.

By this new definition, the unit of temperature is related to a universal constant:

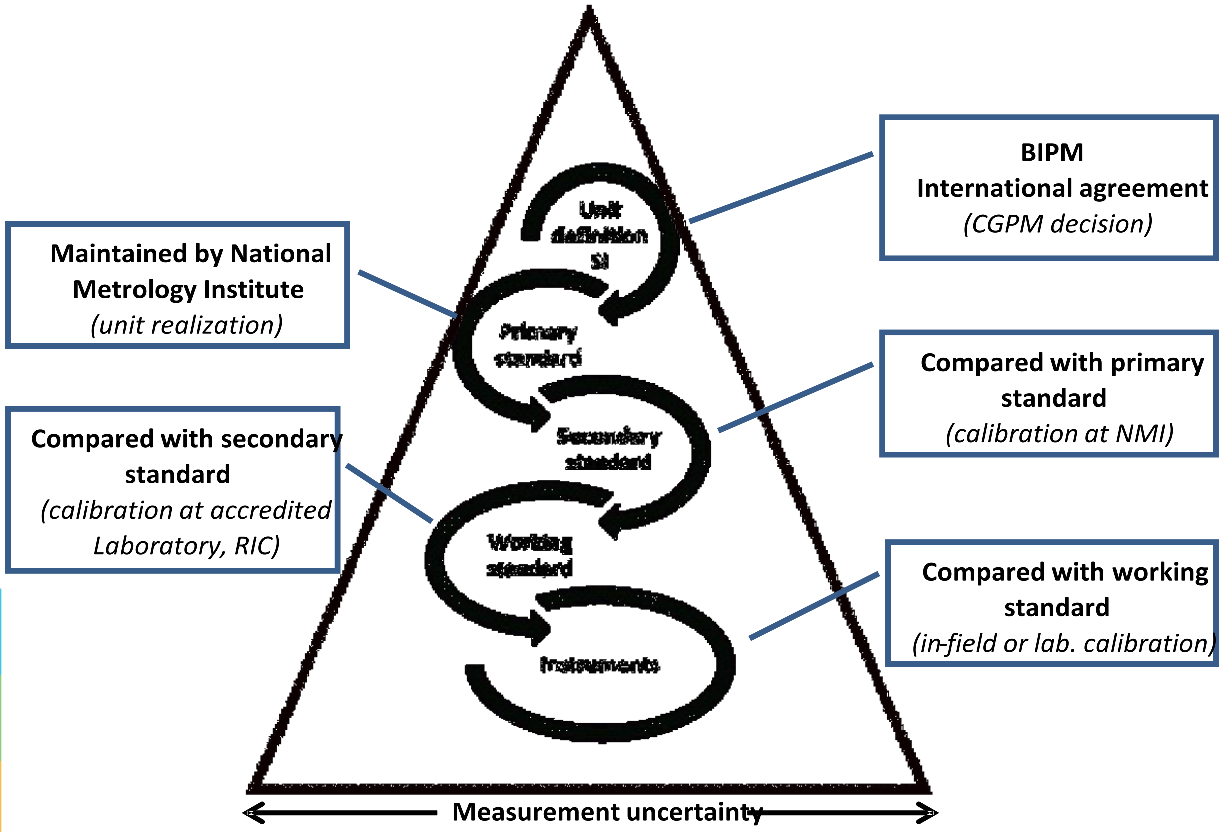
-The purpose of this new definition is to lay the foundations for future improvements by making the kelvin independent of any material element, measurement technique or temperature range in agreement with the overall definition of the seven base units of the SI.

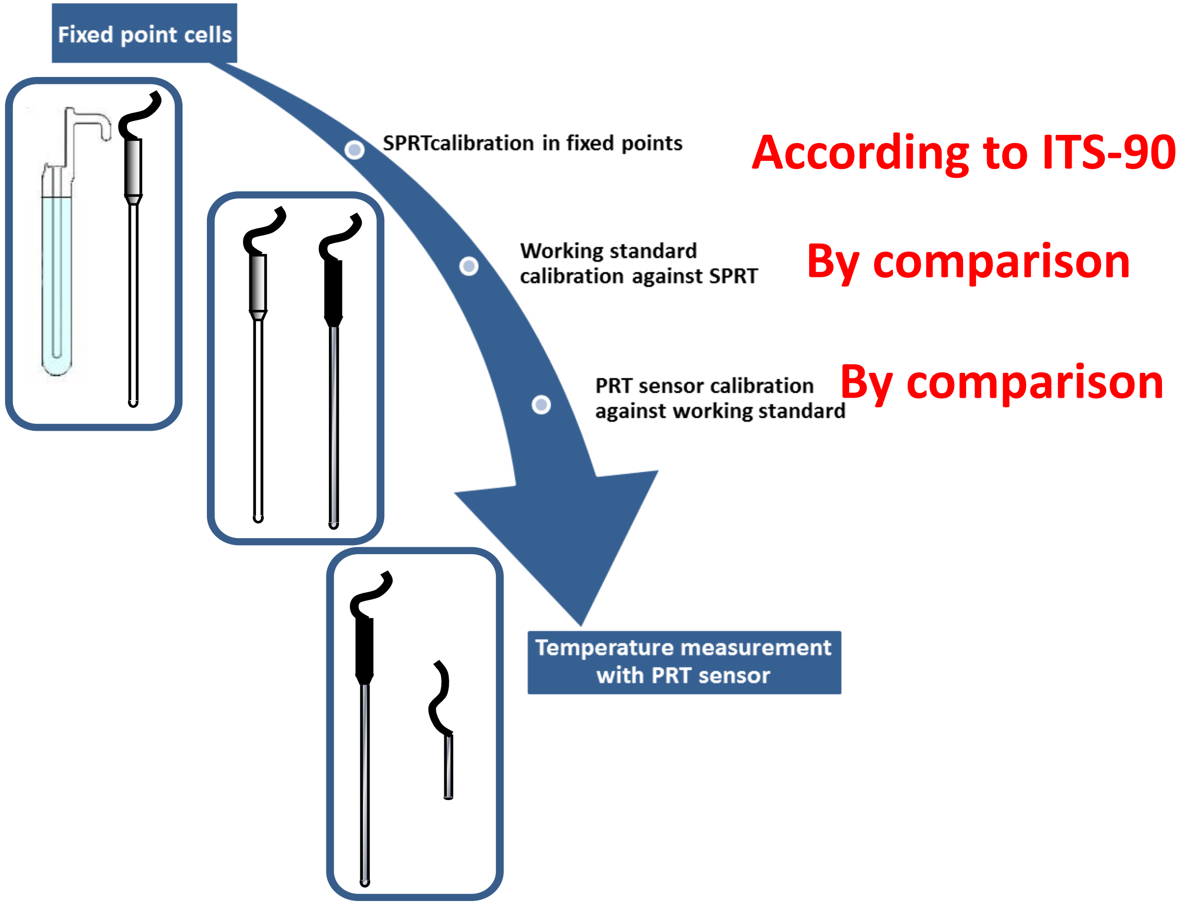
-Although the kelvin redefinition fundamentally modifies the principles and practices of thermometry, the temperature calibrations performed according to ITS-90 are valid and traceable to the SI after the kelvin redefinition.

The unit of Celsius temperature (*t*) is the **degree Celsius**, **symbol °C**, which is by definition equal in magnitude to the unit kelvin; both are units of the International Temperature Scale of 1990 (ITS-90).

$$t/°C = T/K - 273.15$$

Traceability chain in contact thermometry





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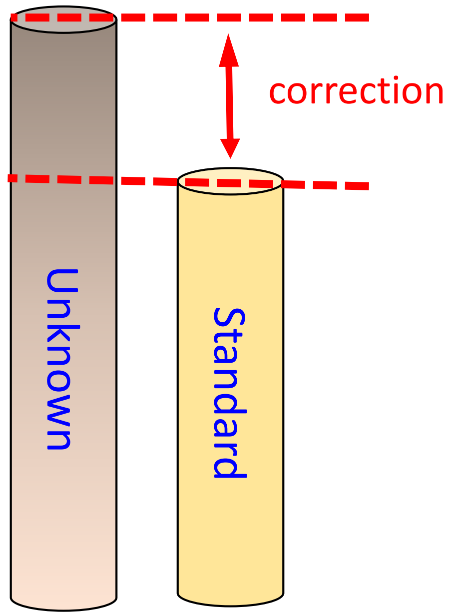
Calibration by Comparison: Definition

-Calibration: operation that, under specified conditions, in a first step, establishes a relation between the **quantity values** with **measurement uncertainties** provided by **measurement standards** and corresponding **indications** with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a **measurement result** from an indication



-The Calibration by the Comparison Method in contact thermometry: the measurements of the thermometer under calibration are compared with the ones of standard thermometers (traceable to the ITS-90) in an isothermal enclosure. **In general, 4 to 5 calibration points covering the calibration range are recommended**.

Correction = value of Standard – reading of instrument under calibration

Error = - Correction

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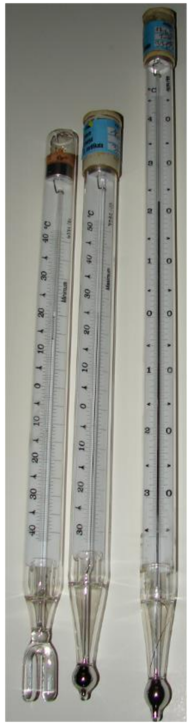
[1] International Vocabulary of Metrology, VIM. www.bipm.org

Calibration by Comparison: Types of thermometers

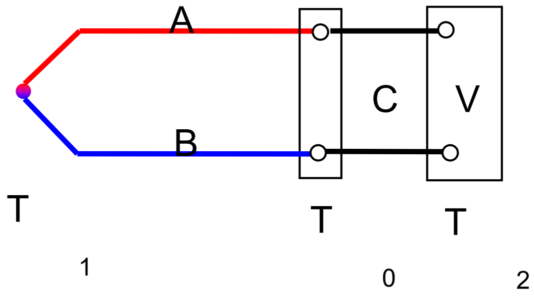
A thermometer is any device which has a measurable property which changes with temperature. To have accurate measurements it is necessary to assure the thermal equilibrium of the thermometer and the measured object/system. In the case of contact thermometers, the equilibrium is reached mainly by heat conduction between the measured object / system and the thermometer.



Liquid-in-glass thermometers:
Expansion of a fluid in a capillary stem with temperature

Platinum resistance thermometers and thermistors:
Change of electrical resistance of metals and semiconductors (thermistors) with temperature

Thermocouple:
Change of the electromotive force along two dissimilar electrical conductors, joined at one end with a temperature difference between their endpoints (Seebeck effect)



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Calibration by Comparison: Uncertainty calculation

The calculation of the calibration uncertainty involves the following steps **in a simplified form**:

1. Express in **mathematical terms** the dependence of the measurand (output quantity: *Y*) on the different input quantities (the different sources of uncertainty: X_i). In the case of a direct comparison of two standards the mathematical model may be very simple: $Y= f(X_i) =X_1+X_2$.

2. Identify and apply all significant corrections if any.

3. Apply the law of propagation of uncertainties to the mathematical model to calculate the combined standard uncertainty:

$$u_c^2(y)=\sum_{i=1}^{N}\left[\frac{\partial f}{\partial x_i}\right]^2\cdot u^2(x_i)$$

Independent input quantities → $Y=X_1+X_2$ → $u_c^2(y) = u^2(x_1)+u^2(x_2)$

$$u_c^2(y)=\sum_{i=1}^{N}\left[\frac{\partial f}{\partial x_i}\right]^2\cdot u^2(x_i)+2\cdot\sum_{i=1}^{N-1}\sum_{j=1+1}^{N}\frac{\partial f}{\partial x_i}\cdot\frac{\partial f}{\partial x_j}\cdot r(x_i,x_j)\cdot u(x_i)\cdot u(x_j)$$

Correlated input quantities → $Y=X_1+X_2$ → $u_c^2(y) = u^2(x_1)+u^2(x_2) + 2r(x_1,x_2)u(x_1)u(x_1)$

Correlation coeficient

4. Calculate for each input quantity X_i the contribution $u(x_i)$ to the combined standard uncertainty:
   - When a probability distribution can be assumed : $u^2(x_1) = \sigma^2(\overline{x})$ (variance of this distribution) (e.g. calibration uncertainty of the standard thermometers).
   - If only upper and lower limits (a_+ and a_-) can be estimated, a rectangular probability distribution can be assumed : $u^2(x_1) = \frac{(a_+ - a_-)^2}{12}$ (e.g. manufacturing specifications, hysteresis, thermoelectric homogeneity…).

5. Calculate the expanded uncertainty U = 2 x $u_c(y)$ If a normal (Gaussian) distribution can be attributed to the measurand a coverage factor *k*=2 corresponds to a coverage probability of approximately 95 %.

Calibration by Comparison: Uncertainty due to reference temperature

For simplicity the uncertainty calibration can be divided in two parts:

- Calculation of the uncertainty of the **reference temperature T_{ref}**, this is the uncertainty corresponding to the measurement system and will be the same no matter what kind of thermometer is under calibration.

- Calculation of the particular contributions of the **thermometer under calibration** which will depend on the type of sensor.

Calculation of the uncertainty of the reference temperature T_{ref}

The uncertainty on the determination of the reference temperature comes from the uncertainties of the measurements performed with the standard thermometer and the stability and uniformity of the isothermal enclosure:

law of propagation of uncertainties

$$T_{\mathrm{ref}} = T_{\mathrm{s}} + \delta T_{\mathrm{stab}} + \delta T_{\mathrm{unif}} \longrightarrow u^2(T_{\mathrm{ref}}) = u^2(T_{\mathrm{s}}) + u^2(\delta T_{\mathrm{stab}}) + u^2(\delta T_{\mathrm{unif}})$$

If two reference thermometers are considered:

$$T_{\mathrm{ref}} = \frac{T_{\mathrm{s1}} + T_{\mathrm{s2}}}{2} + \delta T_{\mathrm{stab}} + \delta T_{\mathrm{unif}} \longrightarrow u^2(T_{\mathrm{ref}}) = \left(\frac{1}{2}u(T_{\mathrm{s1}}) + \frac{1}{2}u(T_{\mathrm{s2}})\right)^2 + u^2(\delta T_{\mathrm{stab}}) + u^2(\delta T_{\mathrm{unif}})$$

When similar contributions for each standard thermometer

$$u^2(T_{\mathrm{ref}}) = u^2(T_{\mathrm{s}}) + u^2(\delta T_{\mathrm{stab}}) + u^2(\delta T_{\mathrm{unif}})$$

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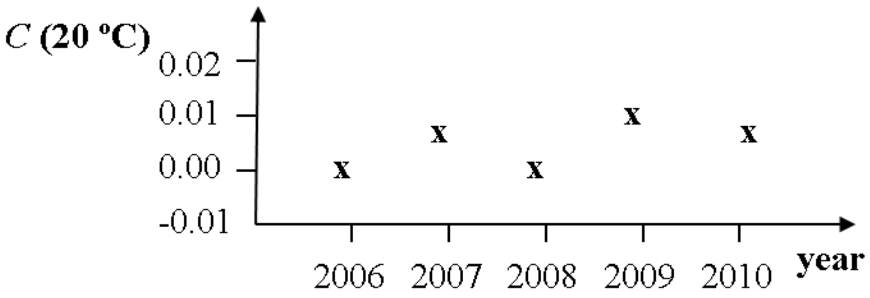
Calibration by Comparison: Uncertainty due to reference temperature

The uncertainty of the measurements performed by the standards $u(T_s)$ has in turn three main components:

- The **calibration of the standards** $u(\delta T_{s,cal})$, whose contribution can be obtained from the calibration certificate dividing the expanded uncertainty reported by the corresponding coverage factor.

- The **drift** of the standards between calibrations $u(\delta T_{s,drift})$. From the known history of the standards a maximum limit for the drift can be assumed and a rectangular probability distribution can be assigned.



Rectangular distribution

$$u(\delta T_{s,drift}) = \frac{0.01\ °C}{\sqrt{3}} \approx 0.005\,8\ °C$$

- The **reading of the standards** $u(\delta T_{s,read})$. This uncertainty will depend of the reading system:
  - If the standard thermometers have an indicator, the contribution will come from its resolution to which a rectangular distribution can be assigned.
  - If a resistance bridge is used then the uncertainty will come from the bridge. In the case the contributions are calculated in Ω, the sensitivity coefficient of the thermometer can be used: 0.4 Ω/°C for a pt-100.

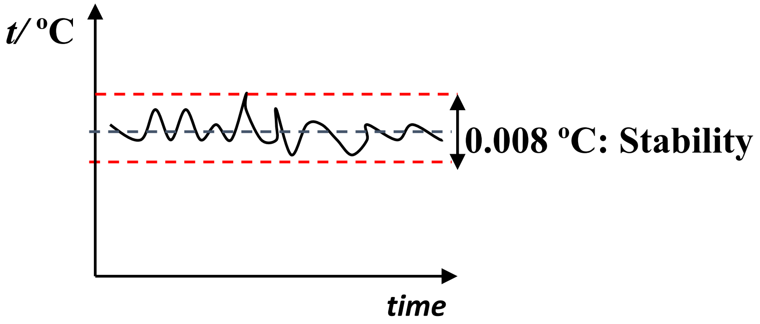
Calibration by Comparison: Uncertainty due to reference temperature

The contributions due to the **stability and spatial uniformity** of the isothermal enclosure should be estimated previously:

-as the maximum temperature variation of the isothermal enclosure in a determined period of time (stability)

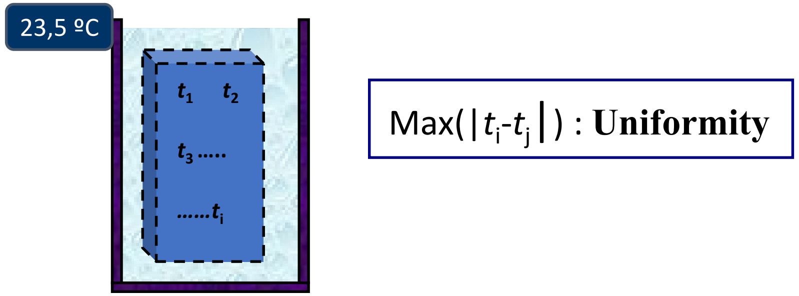
-as the maximum temperature variation of the isothermal enclosure in the useful volume (uniformity).

- in both cases rectangular probability distributions could be assigned.



Rectangular distribution

$$u(\delta T_{\text{stab}}) = \frac{0.008\ ^\circ\text{C}}{\sqrt{12}} \approx 0.002\ 3\ ^\circ\text{C}$$



Rectangular distribution

$$u(\delta T_{\text{unif}}) = \frac{0.01\ ^\circ\text{C}}{\sqrt{12}} \approx 0.002\ 9\ ^\circ\text{C}$$

$$u^2(T_{\text{ref}}) = u^2(\delta T_{\text{s,cal}}) + u^2(\delta T_{\text{s,drift}}) + u^2(\delta T_{\text{s,read}}) + u^2(\delta T_{\text{stab}}) + u^2(\delta T_{\text{unif}})$$

Calibration by Comparison: U due to thermometer under calibration

Calculation of the particular contributions of the thermometer under calibration

The thermometer under calibration has two particular contributions:

- The one coming from its **reading** that will depend of the reading system $u(\delta T_{c,read})$:

-If the thermometer sensor under calibration is connected to an indicator or it is a liquid-in-glass thermometer: resolution + a rectangular distribution

- If a resistance bridge is used to perform the readings, then the uncertainty will come from the bridge. The sensitivity coefficient of sensor needs to be considered:

Platinum resistance thermometers: 0.4 Ω/°C for a pt-100.

Thermistors : the manufacturer specifications should be consulted.

Thermocouples: depends on the type of thermocouples and the temperature range

- The contribution which depends on the particular **type of thermometer**, $u(\delta T_{c,part})$:

Estimated by additional tests: hysteresis or thermoelectric inhomogeneity or depression of zero.

Resistance thermometers → Hysteresis and/or repeatability; → Self-heating

Thermocouples → Thermoelectric inhomogeneity

Liquid in glass thermometers → Hysteresis or depression of zero

In conclusion, the combined calibration uncertainty is:

$$u^2(T_c) = u^2(\delta T_{s,cal}) + u^2(\delta T_{s,drift}) + u^2(\delta T_{s,read}) + u^2(\delta T_{stab}) + u^2(\delta T_{unif}) + u^2(\delta T_{c,read}) + u^2(\delta T_{c,part})$$

Quantity / x_i	Value/ºC	Type of Distribution	Divisor	Contribution to the combined uncertainty/ ºC $u(x_i)$
Contributions due to the calibration system				
Calibration of the standards (information form the calibration certificate)	*Ucertificate/k*	normal	2	$U/2$
Drift of the standards (information from the standards history)	± drift value	rectangular	$\sqrt{3}$	$\frac{drift\ value}{\sqrt{3}}$
Reading system: combination of the uncertainty calibration, drift, resolution and other characteristics of the reading system	Reading system value	normal	1	*Reading system value*
Stability of the isothermal enclosure (previous studies)	± stability	rectangular	$\sqrt{3}$	$\frac{stability}{\sqrt{3}}$
Spatial uniformity of the isothermal enclosure (previous studies)	± uniformity	rectangular	$\sqrt{3}$	$\frac{uniformity}{\sqrt{3}}$
Particular contributions of the thermometer under calibration				
Resolution/Reading system	resolution	rectangular	$\sqrt{12}$	$\frac{resolution}{\sqrt{12}}$
Characteristics of the sensor (hysteresis or thermoelectric inhomogeneity or depression of zero)	± characteristic value	rectangular	$\sqrt{3}$	$\frac{characteristic\ value}{\sqrt{3}}$
			$u(T) =$	$\sum_{i=1}^{n} u^2(x_i)$
			$U(T)$ (k=2) =	$2 \cdot u(T)$

Numerical Example:

Calibration of electronic thermometer (resolution 0.1 ºC) with a pt-100 as sensor (with hysteresis of 0.01 ºC)

Quantity /x_i	Value ºC	Type of Distribution	Divisor	Contribution to the combined uncertainty/ ºC $u(x_i)$
Contributions due to the calibration system				
Calibration of the standards (information from the calibration certificate)	0.020	normal	2	0.010
Drift of the standards (information from the standards history)	0.01	rectangular	√12	0.002 9
Reading system: combination of the uncertainty calibration, drift, resolution and other characteristics of the reading system	0.01	normal	1	0.01
Stability of the isothermal enclosure (previous studies)	0.008	rectangular	√12	0.002 3
Spatial uniformity of the isothermal enclosure (previous studies)	0.01	rectangular	√12	0.002 9
Particular contributions of the thermometer under calibration				
Resolution/Reading system	0.1	rectangular	√12	0.02 9
Characteristics of the sensor (hysteresis or thermoelectric inhomogeneity or depression of zero)	0.01	rectangular	√3	0.005 8
			u_c(°C) =	**0.033**
			***U*(°C) =**	**0.07 (*k* = 2)**

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Thank you.

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