Infrastructure Commission (INFCOM)

Standing Committee on Measurements, Instrumentation and Traceability (SC-MINT)

Expert Team on Quality, Traceability and Calibration (ET-QTC)

Calibration and Measurement Uncertainty

Part-3: Reporting the Uncertainty

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Content

Reporting the uncertainty

The uncertainty of a measurement result is **a single value** obtained by combining all the components of uncertainty into an uncertainty budget, then multiplying by a coverage factor.

- It has no sense to express the uncertainty with more than 2 significant figures.
- The last figures of the measured value must be coherent with the order of magnitude of the uncertainty and vice versa.

20.3 ± 0.1 °C -> Correct
103.245 ± 15 Pa -> Correct
2.346 ± 0.002 g -> Correct

20.3 ± 0.021 °C - > Wrong
103.245 ± 152 Pa -> Wrong
2.346 ± 0.20 g -> Wrong

Reporting the uncertainty

When reporting the uncertainty, include the following information:

- A list of all components of standard uncertainty
- Their degrees of freedom
- The resulting value of U_c
- The components should be identified according to the method used to estimate their numerical values:

  A. those which are evaluated by statistical methods

  B. those which are evaluated by other means.
- A detailed description of how each component of standard uncertainty was evaluated
- The distribution associated to all the components

Combined Standard Uncertainty
Law of propagation

- The **combined standard uncertainty** of a measurement result, symbol u_c, is taken to represent the estimated standard deviation of the result.
- It is obtained by combining the individual standard uncertainties u_i, whether arising from a Type A evaluation or a Type B evaluation.
- Combining standard deviations is often called the *law of propagation of uncertainty* and uses the "root-sum-of-squares" (square root of the sum-of-the-squares) method of combining uncertainty components estimated as standard deviations.

Coverage Factor

- It is common to multiply the combined standard uncertainty, $u_c(y)$, by a factor, k, chosen so that the interval $y \pm k \cdot u_c(y)$ has a specified (larger) probability of containing the true value of the measurand.
- GUM calls product $U = k \cdot u_c(y)$ an ***expanded uncertainty***
- Factor k is called a ***coverage factor*** (often k = 1, 2 or 3)
- The probability that $y \pm U$ contains the true value is called the ***coverage probability***

EXPANDED UNCERTAINTY

(to cover larger probability that the true value falls in the uncertainty interval)

product of a **combined standard measurement uncertainty** and a factor larger than the number one

NOTE 1 The factor depends upon the type of probability distribution of the **output quantity in a measurement model** and on the selected **coverage probability**.

NOTE 2 The term “factor” in this definition refers to a **coverage factor**.

Coverage factor	*k* =	1	1.845	1.960	2	2.576	3
Coverage probability	%	**68.27**	**90**	**95**	**95.45**	**99**	**99.73**

Example of uncertainty budget - Calibration

x_i			
		Distribution	$u(x_i)$ / °C
Components derived from the reference thermometer		Normal	$5.02 \cdot 10^{-3}$
Components derived from measurement system		Normal	$1.27 \cdot 10\text{-}^{2}$
Components derived from meteorological thermometer	repeatibility	Rectangular	$3.47 \cdot 10^{-2}$
	resolution	Rectangular	$4.04 \cdot 10^{-3}$
	reproducibility	Rectangular	$1.40 \cdot 10^{-2}$
	hysteresis	Normal	$2.00 \cdot 10^{-2}$
	$u(x) = (\sqrt{\sum u^2(x_i)})^{½}$		$4.45 \cdot 10^{-2}$
	$\boldsymbol{U(x) = 2 \cdot u(x)}$	$k = 2$	**0.090**

Reporting Uncertainty in field measurement (not calibration)

When completing the measurement uncertainty budget:

1. List all Type B components (instrumental and environmental factors) separate from Type A (such as residual, statistical analysis etc.)
2. Describe all components and report all values in k=1
3. Be sure one of the component is the calibration uncertainty
4. Identify, evaluate and include all environmental effects and quantities of influence
5. Report the distribution associated to every component (normal, rectangular…)
6. Calculate the propagation formula to evaluate the combined uncertainty
7. Multiply by the coverage factor, to obtain the combined uncertainty.

Summary

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Summary of Steps 1/2

1. Define the measurand and construct the mathematical model of the measurement
2. Obtain estimates, x_i, of the input quantities
3. Evaluate the standard uncertainties $u(x_i)$, by Type A or Type B methods, and evaluate the covariance $u(x_i,x_j)$ for each pair of correlated input estimates x_i and x_j
4. Apply the model to evaluate the output estimate, y

Summary of Steps 2/2

5. Propagate the standard uncertainties $u(x_i)$ and covariances $u(x_i,x_j)$ to obtain the combined standard uncertainty $u_c(y)$
6. Optionally, multiply $u_c(y)$ by a coverage factor, k, to obtain an expanded uncertainty, U
7. Report the result, y, with either the combined standard uncertainty, $u_c(y)$, or the expanded uncertainty, U
8. Explain the uncertainty clearly

Some summary concepts

- Measurement uncertainty is a property of a **measurement result**.
- Ideally this would be evaluated for every measurement result.
- There is general acceptance that it is possible to evaluate the uncertainty of a standardised method – and assume this uncertainty applies to future measurements made with that method.
- Need to be sure the uncertainty evaluation is appropriate for all applications of the method – i.e. conditions and scope of the evaluation (and validation) cover the ongoing use.
- Quality requirements within method become important.
- Ideally a method would provide a procedure for a user to evaluate the measurement uncertainty of results they have obtained, and the results of validation of the method uncertainty.

Uncertainty is also a matter of personal involvement:

- different operators can compile an uncertainty budget in different ways;
- different reasons can motivate moving Type A components into Type B;
- different solutions can be adopted to minimize the uncertainty.

The measurement procedures shall describe how each uncertainty contribution is determined.

End of Uncertainty Unit

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