## Title: How do we teach the Sub-Tropical Jetstream?


#### Abstract

The Sub-Tropical Jetstream (STJ) is the most powerful wind system on Earth. It is confined to the upper troposphere at $30^{\circ}$ latitude in both hemispheres, and the westerlies within it can reach speeds greater than 150 kt during winter.

In their classic 1969 text, Palmén and Newton state that the STJ "is generated as a result of the systematic poleward drift of air in the upper branch of the Hadley cell of the general atmospheric circulation, with partially conserved absolute angular momentum. In accord with this concept, the subtropical jet stream is located near the poleward boundary of this cell."

Wiin-Nielsen and Chen (1993) note the importance of the STJ, and state that "since Rossby and his collaborators initiated a pioneer study of several aspects of subtropical jet streams, numerous efforts have been made to explore their structure, their relation to other weather systems and the mechanisms that create and maintain them. However, we still lack a satisfactory mechanism explaining the existence of subtropical jet streams." Indeed, most textbooks today provide good coverage of the characteristics of STJs but seem to cover lightly the mechanisms for their existence.

Five years later, Gordon et al. (1998) describe a mechanism that can, at least partially, explain both the winter STJ and the existence of the subtropical anticyclones. Their explanation relates to the properties of upper-air trajectories departing the tropics meridionally and being deflected by the varying Coriolis acceleration. Their explanation is founded on two principles: - the rate of change of the Coriolis parameter is a maximum at the equator, even though its magnitude is zero there, and - a very small acceleration acting on a parcel of air for a long duration results in a large speed and displacement of the parcel.

These principles, although basic in nature, have profound consequences in terms of the general circulation of the atmosphere. Students who are new to meteorology may not appreciate their significance.

This paper explores the mechanism put forward by Gordon et al. to explain the existence of the STJ, and how aspects of it can be used to help students understand some of the physical processes occurring in the atmosphere.


## Discussion paper

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## 1. Introduction

At a previous CALMet conference (Melbourne, 2017) I presented a paper titled "Reflections on student-centred learning". The aim was to share ideas drawn from practical experience on how to enhance the learning and understanding of students. In this paper I will extend the topic by sharing more pedagogical ideas on the topic of the subtropical Jetstream (STJ).

In the abstract of the current paper I stated the importance of the STJ and presented a brief historical account of attempts to explain the mechanisms driving it. Here I want to dig deeper into the nature of these mechanisms. There will be discussion of the processes that are key to understanding the STJ and some aspects that instructors should, I believe, consider if they intend to teach about this feature of the upper atmosphere.

The presentation is given here in a way that should appeal to both mathematics and physics students. The aim is to get students thinking, and to help and encourage them to make sense of three-dimensional concepts in geophysics. We first need to introduce the so-called Coriolis parameter, $f$.

## 2. The Coriolis parameter

Many atmospheric phenomena are influenced by $f$. Its definition and properties are often new to students starting out in geophysics. So let's take a closer look at it and experiment with ways to describe it.

- We define the Coriolis parameter $f=2 \Omega \sin \phi$, where $\Omega$ is the Earth's rotation rate (1 revolution per day) and $\phi$ is latitude ( $0^{\circ}$ at the equator, $90^{\circ}$ at the N Pole and $-90^{\circ}$ at the S Pole).
- In Tangential-Cartesian coordinates, we define $\beta=d f / d y=2 \Omega \cos \phi / a$, where $a$ is Earth's radius (about 6500km).

Below is a graph of $f$ and $\beta$ with respect to latitude, presented in a conventional way:


Fig. 1 - Variation of $f$ and $b$ with latitude.

The plot below is another way of showing the variations of $f$ and $\beta$ with latitude. What do you notice about the changes in these quantities?


Fig. 2 - Another way of looking at the variation of $f$ and $\beta$ with latitude.

In my opinion Fig. 2 gives a better and more memorable understanding of $f$ and its properties than Fig. 1. Do you agree?

Geostrophic balance is achieved when the horizontal Coriolis force and pressure gradient force are in balance. The Coriolis force acts at right angle to the wind, and its magnitude is the product of the Coriolis parameter and the wind speed. So the nature of $f$ is critical to geostrophic balance and whether it can even be achieved.
$f$ has unit of $\mathrm{s}^{-1}$ and is a maximum positive at the North Pole, 0 at the equator, and a maximum negative at the South Pole. What does the unit "per second" actually mean? If we think of the Coriolis parameter in terms of its definition, then it is the product of $\Omega$, in radians per second, with dimensionless quantities. If we think of the Coriolis parameter as the Earth's vorticity, then the Coriolis parameter is the (vertical component of) curl of a velocity - it represents a change of velocity with distance so it has units of $\mathrm{ms}^{-1} \mathrm{~m}^{-1}$.

From the above we can see that it's reasonable for many purposes to keep the Coriolis parameter constant in temperate or polar latitudes, but not in tropical latitudes. That's despite the fact that $f$ is small in the tropics!

## 3. Geostrophic Adjustment

Most basic books and online resources that present geostrophic balance depict the process of air adjusting to this balance as ending with the wind becoming exactly geostrophic. As Gordon et al. (1998) note, this is far from the whole truth. Here's a quote from their book about how the wind adjusts to geostrophy:

In some non-mathematical descriptions of the geostrophic adjustment process, ... it has been said that the wind begins to flow across the isobars but eventually lines up along the isobars. At this stage geostrophic balance is achieved. This description only tells half the story. The simple mathematics tells us a great deal more of the detail of what must happen. Certainly, the geostrophic adjustment process depends on the length of the inertial period. In low latitudes where the inertial period is long, geostrophic adjustment is unlikely to occur because the pressure gradient will not stay constant for such long periods ( Gordon et al, 1998, p. 121).

Gordon et al. go on to explain this in their book, including how the wind-speed will fluctuate during adjustment to up to twice the geostrophic speed.

A discussion question: Is it justifiable to gloss over the whole truth and tell only half the story? What if students go further in their studies and the misleading story is never corrected or becomes rooted?

## 4. Description of the STJ

Have a look at the images of monthly mean u-winds. These have been computed from NCEP reanalysis data. What do you notice about the location of the strongest winds at the 250 hPa level?


Fig. 3 - Long-term mean $u$-winds (zonal wind components) at 250 hPa , about 10km above the Earth's surface ( $\mathrm{ms}^{-1}$ ) for January (top) and July (bottom). Courtesy NOAA/ESRL physical sciences division.

We can see that there is a wind maximum in the winter hemisphere at $30^{\circ}$ from the equator, reaching speeds greater than $50 \mathrm{~ms}^{-1}$ ( 100 knots).

The diagram below shows the typical locations of the main jet-streams. It is drawn for the Northern Hemisphere but is also applicable to the Southern Hemisphere.


Fig. 4 - Schematic cross-sectional model of the troposphere. Heavy solid line: dynamic tropopause; shaded area: stratosphere; dotted lines: main frontal zones; thin dashed line: $40 \mathrm{~ms}^{-1}$ isotach indicating climatological jet-streams; Ja = Arctic or Antarctic jet, Jp = Polar Front jet, Js = subtropical jet. From Shapiro, Hampel \& Krueger (1987).

## 5. Air parcel trajectories at low latitudes

Gordon et al. develop their argument by considering the trajectories of air parcels at tropical (low) latitudes (Gordon et al., 1998, p123). They allow the Coriolis force to vary with latitude, using westeast oriented isobars as for the geostrophic case, and let air parcels start from rest from different latitudes at $5^{\circ}$ intervals. Numerically computed upper-tropospheric trajectories of this kind are shown in the plot below. Although the trajectories are theoretical, the assumed values of atmospheric elements are realistic, including the meridional pressure gradients. See the Excel spreadsheet "Replicating_Gordon_et_al1998_Fig_10-3\&10-4.xlsx" accompanying this paper for details.

Note the convergence of the trajectories between those starting from the equator and those starting a few degrees away from the equator. It is also found that the air parcels originating nearer the equator achieve a greater speed at $20^{\circ}$ to $25^{\circ} \mathrm{N}$. See the spreadsheet for details, which allow students to change settings and get a feel for the quantities involved.

## Trajectories of air-parcels starting from rest at 5 deg intervals from the equator in realistic $\mathrm{N}-\mathrm{S}$ pressure gradient



Fig. 5 - Numerically generated horizontal air parcel trajectories, for air parcels starting from rest at $5^{\circ}$ intervals from $0^{\circ}$ to $15^{\circ} \mathrm{N}$. The pressure gradient force is assumed to act from south to north, and the Coriolis parameter is varying with latitude.

Gordon et al. note that Fig. 5 gives
a simple, but interesting and useful, interpretation of the behaviour of the atmosphere in the tropics in the real world. In [the Figure] the gradient is small but of the order of magnitude which may exist in winter at upper levels poleward of the equator in one or the other hemisphere. Both the winter subtropical jet-stream and the existence of the winter subtropical anticyclones can, at least partially, be explained by the pattern of trajectories appearing in this figure. Note the convergence of the trajectories where they become parallel to the isobars...
(Gordon et al., 1998, p. 124)

They then note that upper-level convergence contributes to the building up of surface anticyclones. If the Figure above is a possible example of the behaviour of the troposphere in the tropics it suggests the presence of the subtropical anticyclones and associated subsidence in subtropical latitudes (about $30^{\circ} \mathrm{N}$ and S ).

## 6. The effect of cross-equatorial flow

The Swedish meteorologist Anders Persson noted the work of Gordon and Shaw (1954) and suggested the following explanation for the STJ:
"The 8-term... seemed to play an unexpected role. Air parcels coming from the equator, or a band close to the equator where the Coriolis effect is nil or very small, at first moved poleward fairly unaffected by any deflection. It was only when they passed $10^{\circ} \mathrm{N}$ and got into "Coriolis infected" latitudes that they began to turn east.

Those of these air parcels which had started south of the equator had by then travelled 10-20 longitude degrees (circa 1000-2000 km) and had had time to accelerate to rather high velocities. Reaching their maximum latitude in the subtropics their speed would, according to a Table II in the [Gordon and Shaw] paper, have reached $81-91 \mathrm{~m} / \mathrm{s}$.

On the other hand, air starting at $10-20^{\circ} \mathrm{N}$ would immediately move into "Coriolis infected" areas, be deflected and be less, by a factor cosine (lat.), accelerated by the poleward acceleration and finally only reach speeds of $35-51 \mathrm{~m} / \mathrm{s}$.
A. Persson (2020)

As Persson noted, in the Northern Hemisphere winter, when the Hadley cell is displaced southwards, the air aloft accelerates northwards and travels a longer distance over a longer duration than in the opposite season. Although the acceleration is small, it acts over a long duration. The resulting largescale geostrophic adjustment sets the poleward limit of the Hadley circulation.

To appreciate the effect of a small acceleration acting over a long duration, refer to the spreadsheet "effect_of_small_accel_over_long_duration.xlsx". This spreadsheet illustrates how a very small acceleration acting over a long duration results in a very large speed. The student may insert different durations and accelerations into the spreadsheet to get a feel for how these quantities change.

As an example of this effect, suppose a body starts from rest and is subject to an acceleration of 0.7 $\mathrm{mm} \mathrm{s}^{-2}$. Ignoring other effects such as friction, after 24 hours the body will reach a speed of $60 \mathrm{~ms}^{-1}$ (118 knots) and will have travelled 2,613 km.

Fig. 6 extends the Fig. 5 by introducing air parcels that start from rest south of the equator, and accelerate northwards initially with a weak Coriolis deflection to the left of the motion, then with an increasing deflection to the right of the motion. Air parcels originating south of the equator accelerate for a longer duration than those starting to the north.


Fig 6 - Numerically generated horizontal air parcel trajectories, for air parcels starting from rest at $5^{\circ}$ intervals from $10^{\circ} \mathrm{S}$ to $10^{\circ} \mathrm{N}$. The pressure gradient force is assumed to act from south to north, and the Coriolis parameter is varying with latitude. This situation corresponds with the Northern Hemisphere winter, when the Hadley cell is displaced southwards. Compared with the previous figure, in this case the air parcels originating south of the equator achieve a much greater speed.

## 7. Other contributions

Before we conclude, we must note that other processes are likely driving or influencing the STJ. For example, Wallace and Hobbs (2006) attribute the subtropical high-pressure belt to mid-latitude processes:

Baroclinic waves drive their own weak northern and southern hemisphere mean meridional circulation cells, referred to as Ferrel cells, characterised by poleward, frictionally induced Ekman drift at the latitude of the storm tracks ( $\sim 45^{\circ}$ ), ascent on the poleward flank, and descent on the equatorward flank... Hence, with the spontaneous development of the baroclinic waves, the Hadley cells withdraw into the tropics, and a region of subsidence develops at subtropical $\left(\sim 30^{\circ}\right)$ latitudes. These regions of subsidence coincide with the subtropical anticyclones... (Wallace and Hobbs, 2006)

Williams (1988) created computer simulations that showed how the location and strength of the STJ
would vary with different values of $\Omega$, Earth's rotation rate. A faster rotation resulted in a weaker jet closer to the equator, and a slower rotation resulted in a stronger jet nearer the poles.

It seems that the observation by Wiin-Nielsen and Chen (1993) holds some truth to this day, that we still do not have a full explanation for the existence of the STJ.

## 8. Summary

In this paper I have discussed the mechanisms behind the Sub-Tropical Jetstream, and presented ways of thinking about these mechanisms. Instructors are encouraged to follow my lead. Rather than simply provide definitions, facts and traditional diagrams, they can teach in an interactive and exploratory way, allowing the student to get a sense of joint discovery as they learn about this important atmospheric feature.

## 9. References

Gordon, A.H and P. M. Shaw, 1954: Application of Particle Dynamics to Derive a General Circulation in Low Latitudes. Archiv fur Meteorologie, Geophysik and Bioklimatologie, 6, 319-333

Gordon, A., W. Grace, R. Byron-Scott, and P. Schwerdtfeger, 1998: Dynamic Meteorology: A Basic Course. Hodder Arnold Publication, [See Chapter 10.4]

Palmén, E., and C. Newton, 1969: Atmospheric Circulation Systems: Their Structure and Physical Interpretation. Academic Press, [See p. 212, 214]

Persson, A., 2020: Personal communication, from an AMS Community Open Forum
Shapiro, M.A., T. Hampel, and A.J. Krueger, 1987: The arctic tropopause fold. Monthly Weather Review, 115, 444-454

Wallace, J.M and P. V. Hobbs, 2006: Atmospheric Science: An Introductory Survey, Second edition. Academic Press (Elsevier)

Wiin-Nielsen, A., and T-C. Chen, 1993: Fundamentals of Atmospheric Energetics. Oxford University Press, [See p. 151]

Williams, G. P., 1988: The dynamical range of global circulations - II. Climate Dynamics, 3, 45-84

