Derivation of Unit Hydrograph

PRINT VERSION MODULE

Derivation of Unit Hydrograph (UH)

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MODULE OBJECTIVES

- To understand the type of hydro-meteorological records needed for UH derivation
- To familiarize with steps involved in extraction of Direct Runoff Hydrograph (DRH) from flood hydrograph and rainfall excess from rainfall hyetograph
- To determine UH by different approaches
- To learn how to convert a UH of given duration to another duration

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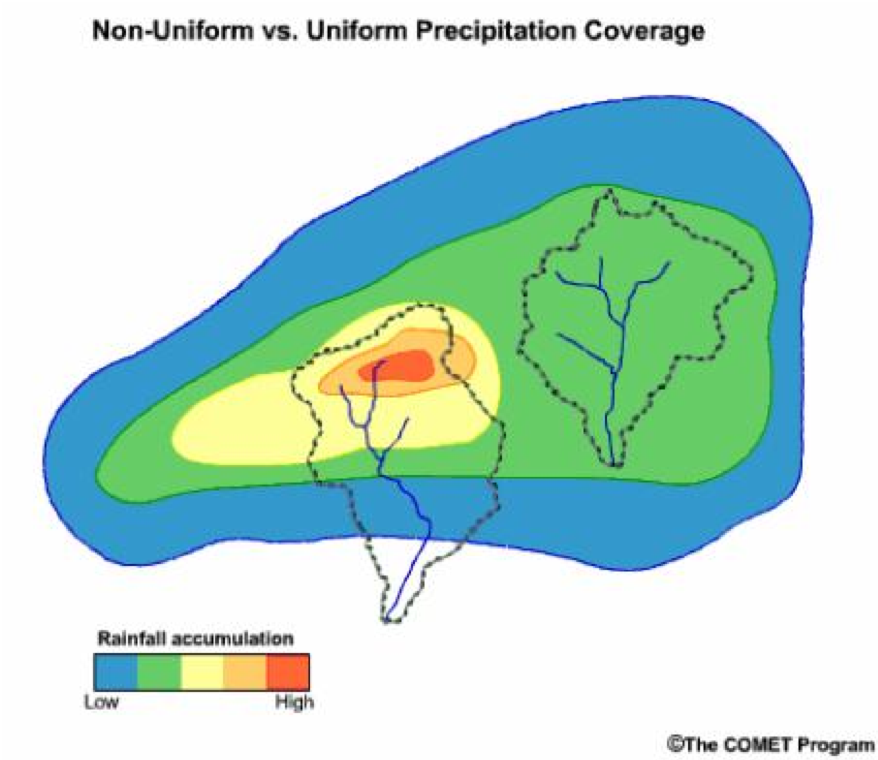
UNIT HYDROGRAPH

The Unit Hydrograph (UH) is the simplest but at the same time a very powerful tool for hydrological analysis in general and flood forecasting in particular.

The unit hydrograph may be defined as the direct runoff (outflow) hydrograph resulting from one unit of effective rainfall, which is uniformly distributed over the basin at a uniform rate during a specified period of time known as unit time or unit duration. The following paragraphs make this statement still clearer.

- ✓ Effective rainfall should be uniformly distributed over the basin, i.e. if there are five rain gauges in the basin, which represent the areal distribution of rainfall over the basin, all the five rain gauges should record for almost same amount of rainfall during specified time. A watershed shown on the right here fully marks this stipulation, while converse is true in respect of left one.



- ✓ In addition, effective rainfall should be at a uniform rate during the unit duration. If the average rainfall over a particular basin during 6 hour is 126mm, a unit hydrograph of 6 hours duration can be derived only if the intensity of rainfall is more or less 21 mm/hour over 6 hours. If the same amount of rainfall is distributed with varied intensity, the unit hydrograph cannot be precisely estimated by simple method.

The unit quantity of effective rainfall is normally taken as 1mm or 1cm; and the outflow hydrograph is expressed by discharge in cumec. The unit duration may be of 1-hour duration or more, depending upon the size of the catchment, storm characteristics and operational facilities. *However, the unit duration cannot be more than the time of concentration or basin lag or period of rise.* The concept of time of concentration has been covered in detail later in the chapter.

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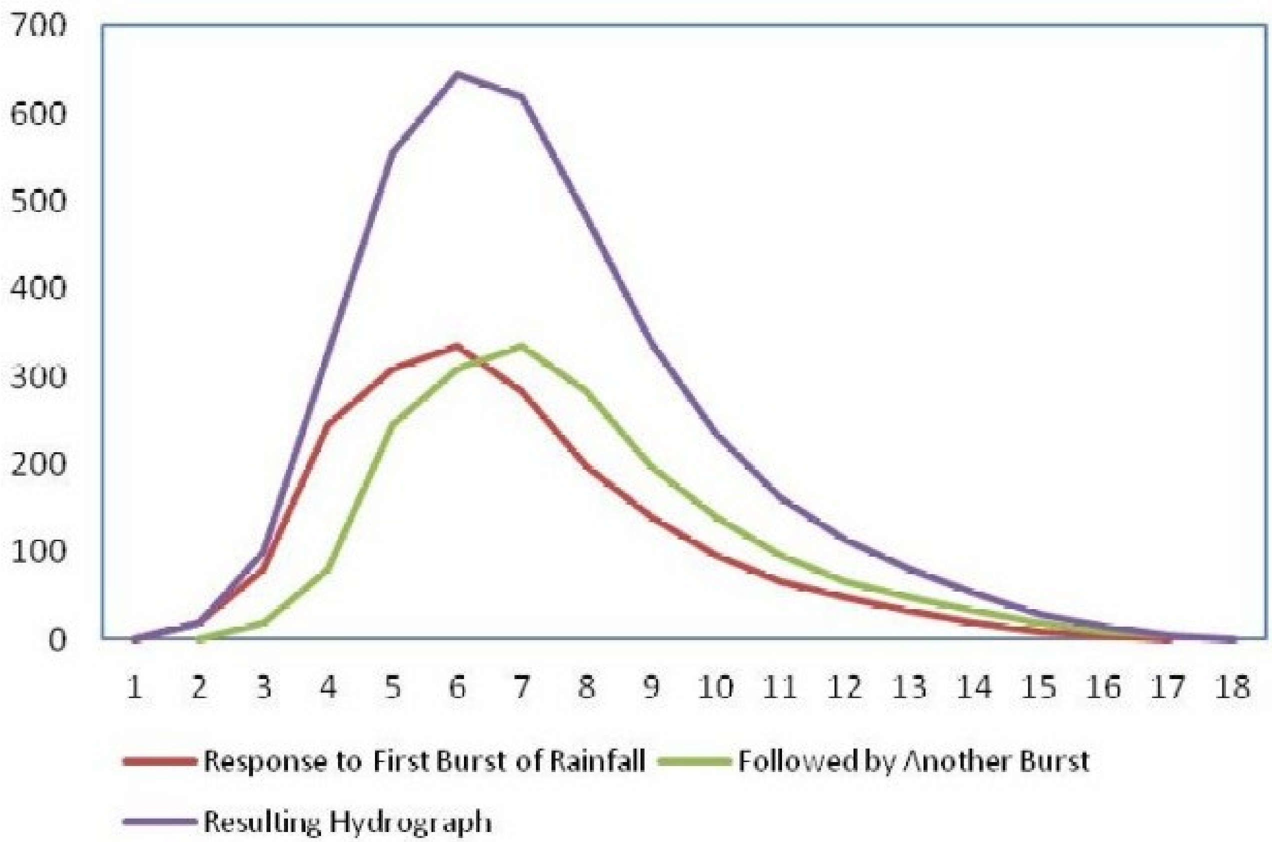
ASSUMPTIONS IN UNIT HYDROGRAPH THEORY

The following are the basic assumption in the unit hydrograph theory:

- The unit hydrograph theory assumes the principle of time invariance. This implies that the direct runoff hydrograph from a given drainage basin due to a given pattern of effective rainfall will be always same irrespective of the time, i.e. even if the basin characteristics change with season etc., the unit hydrograph remains the same.
- Unit Hydrograph theory assumes the principle of linearity, superimposition or proportionality. It means that:
  - If the ordinates of a unit hydrograph of say 1 hour duration are 0,1,6,4,3,2,1,0 units respectively, the effective rainfall of 2 units falling in 1 hour will produce a direct runoff hydrographs having ordinates of 0,2,12,8,6,4,2,0 units.
  - Secondly, if the effective rainfall of two units occurs in 2 hours, i.e. 1 unit per hour, the direct runoff hydrograph ordinates will be obtained by summing up the corresponding ordinates of the two unit hydrographs as shown here.



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DERIVATION OF UNIT HYDROGRAPH (UH)

Selection of a particular UH derivation techniques primarily governed by three factors:

- The UH is best derived from the observed hydrograph resulting from a storm which fulfils the two basic conditions i.e., the rainfall is more or less uniformly distributed over the basin and has a reasonably uniform intensity. Such a hydrograph will generally form an isolated peak.

- In case, such a hydrograph is not available, the UH has to be derived from the analysis of an observed multi-peaked flood hydrograph resulting from several spells of rainfall of varying intensities.
- When the observed discharge and rainfall data at short interval are not available, the synthetic UH is derived with the help of basin characteristics. .

In this module, UH derivation under all three conditions has been illustrated step-wise. At the end of this module, it is expected that reader would be able handle UH assignment independently.

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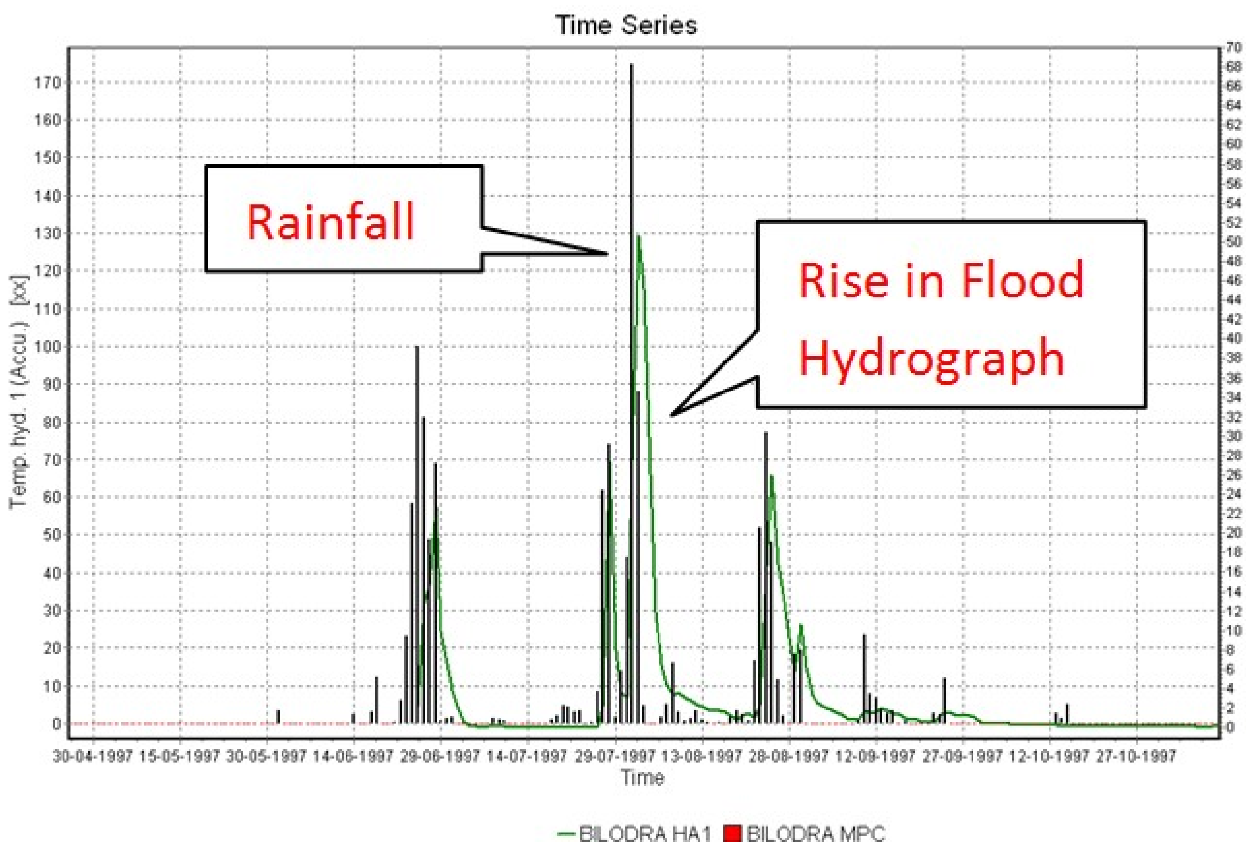
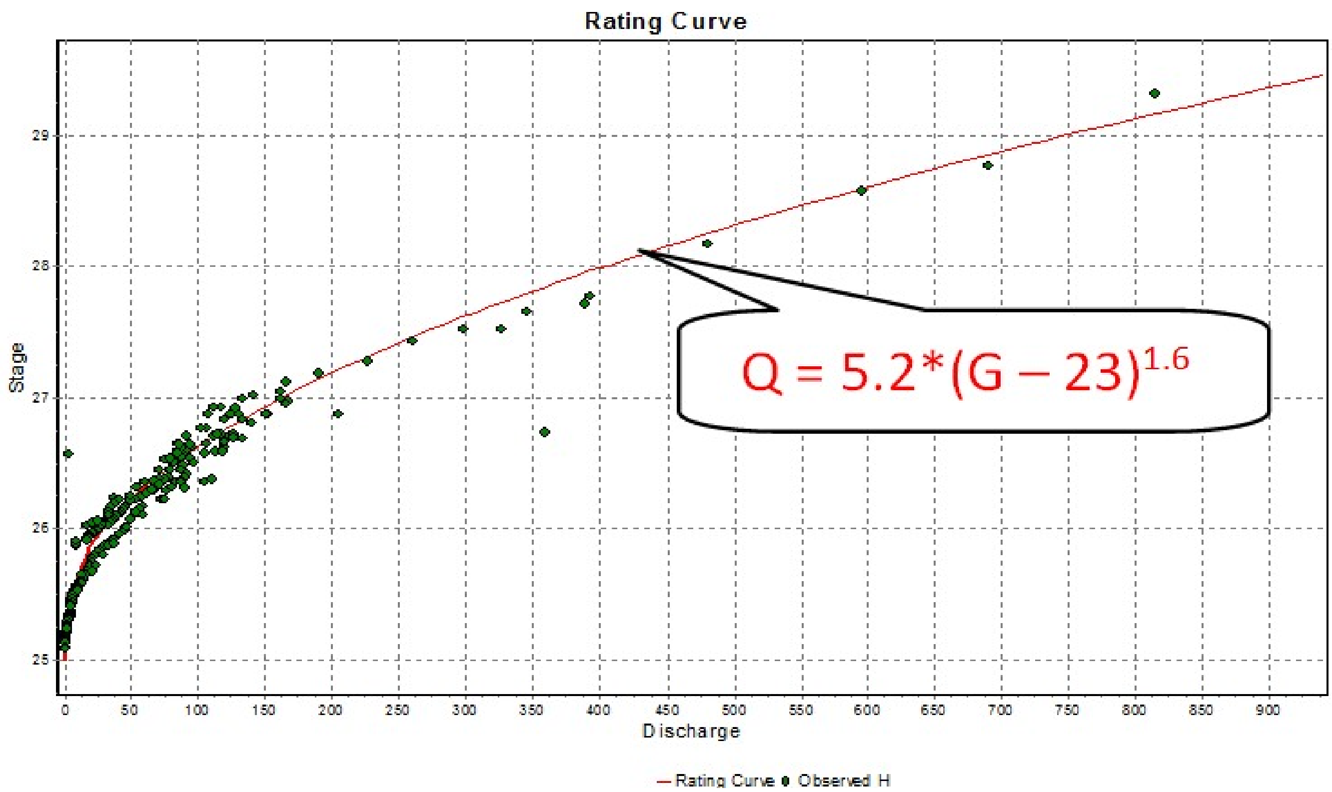
CASE-I - UNIT HYDROGRAPH FROM A HYDROGRAPH WITH ISOLATED PEAK

The steps involved in derivation of UH from the analysis of the flood hydrograph with a single peak are as follows;

Inspect discharge records at watershed outlet and corresponding rainfall events to identify events exhibiting isolated, well defined and single peak with considerable run-off volume. Pick up as many sets of such records as available. A plot displaying rainfall and corresponding rise in flood hydrograph, such as here, can help selection of records.



Note - A no-break/continuous discharge series, as shown in the plot, is developed by transforming hourly river stage (also water level) into discharge with the help of rating equation. Rating equation used for this purpose must be developed for the period to which flood event belongs to.



A rating equation/curve is an equation that relates discharge with water level observed at a site, and is mathematically expressed as

$$\mathbf{Q = c * (G - G_o)^n}$$

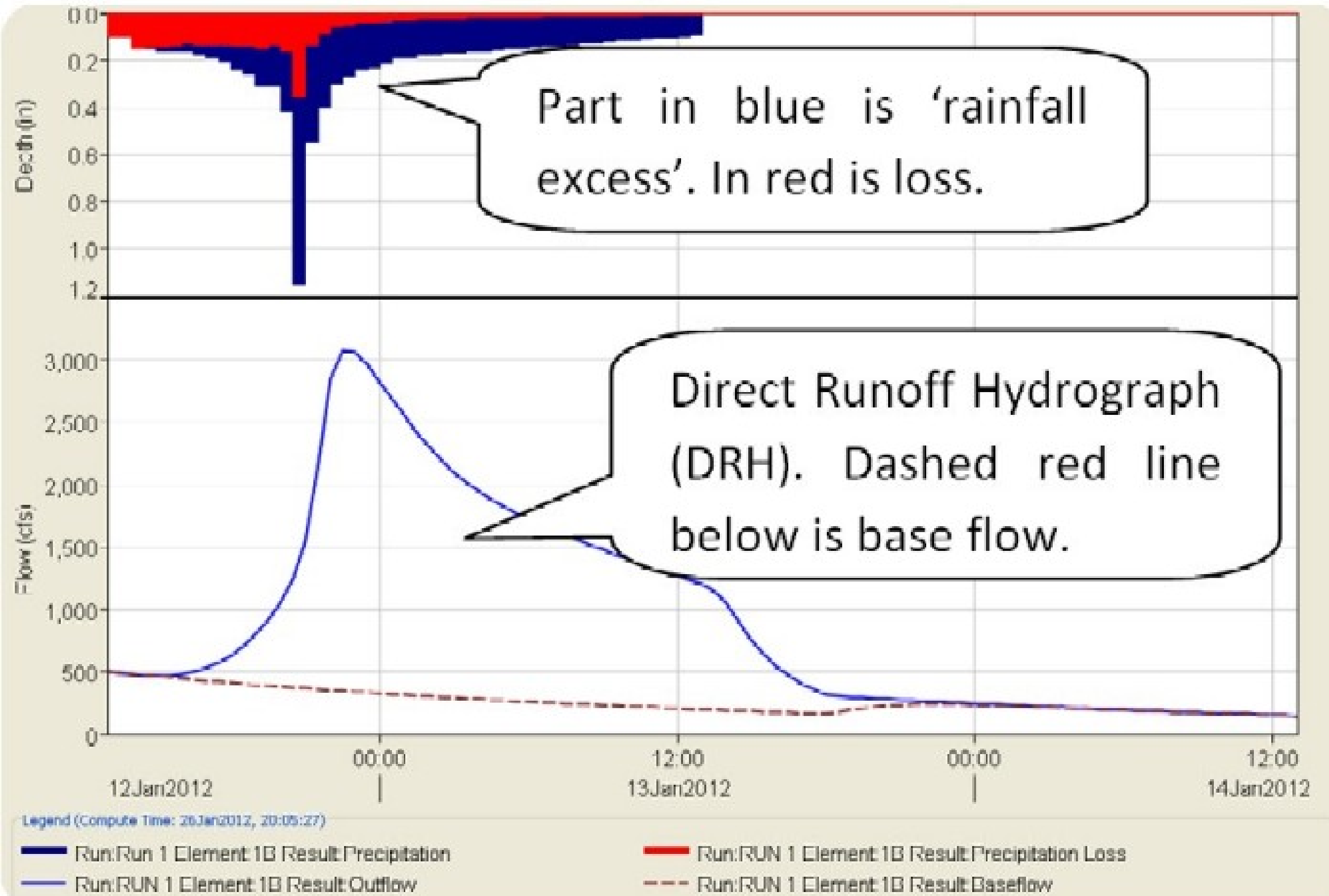
Where, c, G_o; & n are constants; and G is water level.

While gathering information as listed above, it is recommended that

- Storms with rainfall should have been active for duration of around 20 to 30 % of ***basin lag***. Various studies estimate basin lag as 50-75% of T_c, time of concentration. Later part of this chapter describes ways to estimate T_c,
- Storms should have generated rainfall excess between 1 cm and 4.5 cm.

2. A flood hydrograph is a basin (catchment) response driven by occurrence of rainfall event plus contribution of base flow. Had there been no rainfall over the basin, 'Bulge' (rise) in flood hydrograph would have not appeared. Secondly, all water that falls over a catchment does not reach the river/stream because of 'losses'; and only a fraction of it contributes to this 'Bulge'. This

bulge is termed as Direct Runoff Hydrograph (DRH). The part that reaches the stream is called as 'Rainfall excess'. Hydrologist seeks to develop a relationship between 'rainfall excess' and DRH. Apparently, the next step is separation of base flow from flood hydrograph to compute volume of DRH. Following are couple of methods outlined for separation of base flow.

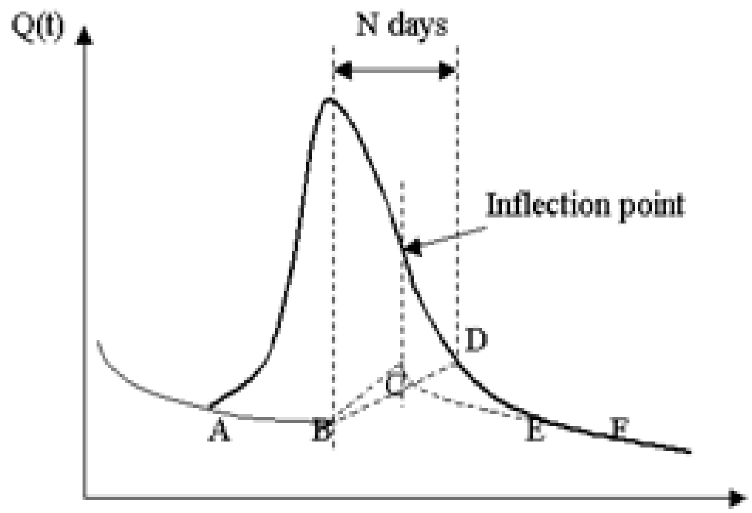


Fixed base method (A-B-D)

This method suggests the extension of the base flow line along its general trend before the rise of the hydrograph up to a point B directly below the runoff hydrograph peak. From B, a straight line BD is drawn to meet the hydrograph at point D, which is N days away from B in the time scale. 'N' is determined by an empirical relation by Linsley as:

$$N \text{ (in days)} = 0.83\, A^{0.2}$$

Where, A is the area of the drainage basin in square kilometers.



Variable Slope Method (A-B-C-E)

This method requires identification of two additional points on the recession limb of hydrograph - one is inflection point; while the other is point E. At inflection point, curve changes its concavity. This point also indicates end of surface flow to river. This point beyond, discharge is a combination of interflow (also called as sub-surface flow) and base flow. After a while, interflow also ceases; and only base flow remains in the river. The 'E' suggests this stage. Once, these two points are located on the graph, a line from 'E' is drawn backward to meet a vertical line from inflection point. A line A-B-C-E divides the DRH and base flow.

Nevertheless, for flood studies, the base flow component is rather insignificant and hence does not influence the magnitude of peak runoff substantially. Therefore, inaccuracies involved in separation of base flow are not crucial in overall flood studies.

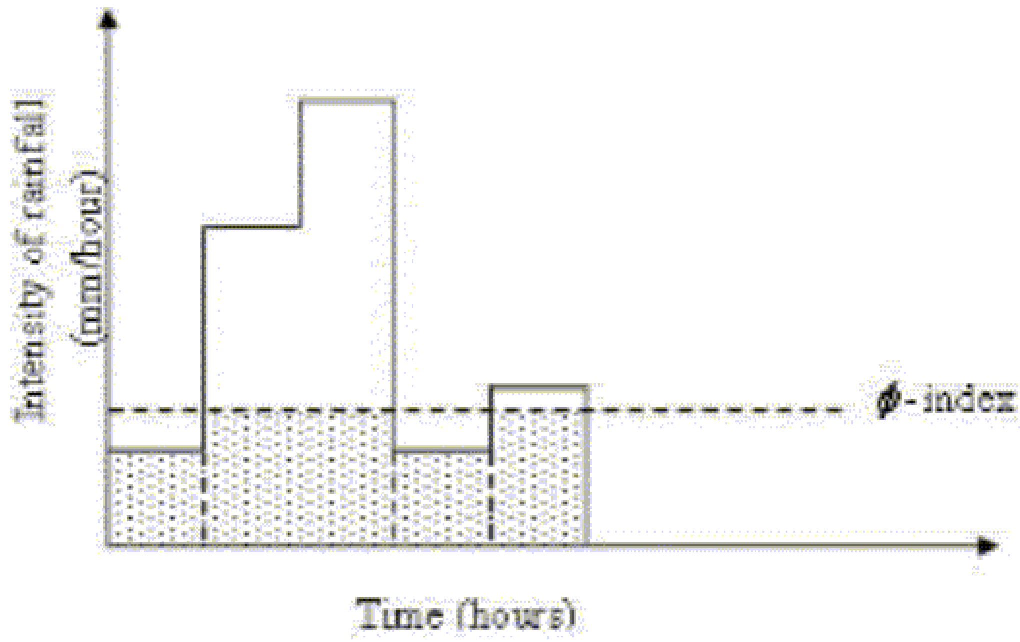
3. Computation of direct runoff hydrograph ordinates by deducting base flow ordinates from that of the corresponding observed flood hydrograph.

4. Scanning and analysis of the rainfall data of all rain gauge stations in and around the basin with a view to;

- Obtaining areal rainfall over the catchment by appropriate methods, such as Thiessen Polygon or Isohyetal technique, and
- Estimating phi-index. Volume of DRH equals the product of catchment area and rainfall excess over the basin. This simple analogy helps us estimate depth of rainfall excess.

Rainfall Excess = Volume of DRH / Catchment Area

A gap between effective rainfall and averaged rainfall points to losses. Here, in the plot, ? - index (also known as loss rate) is drawn in a manner that partitions hyetograph into two parts- lower indicates losses, while upper rainfall excess.



5. As DRH is a consequence of given rainfall excess, say 'x' unit. Estimation of the ordinates of the UH is obtained by dividing the ordinates of direct runoff hydrograph by 'x' rainfall excess.

- **How to fix duration of UH?**

A plot shown here exhibits duration of rainfall as 5-unit, of which only during 3-unit duration rainfall exceeds 'loss rate'. Thus, for this case, UH duration is a 3-unit.

6. This process is repeated for all records picked up for this purpose.

7. It is highly probable that UHs derived for more than one record may differ in duration of excess rainfall. They need to be converted to an identical duration before attempting step 8. A discussion on conversion of UH duration has been added toward the end of this module.

8. All such UHs are eventually averaged. For this, first peaks, Qp, of all UH are averaged to give Qp, followed by time to peak, tp and time base, Tb of UHs. All other ordinates are adjusted in such a way that total run-off volume of UH equals the product of 1cm/mm and catchment area.

More discussions on conversion of UH duration and averaging of UH have been added toward the end of this chapter.

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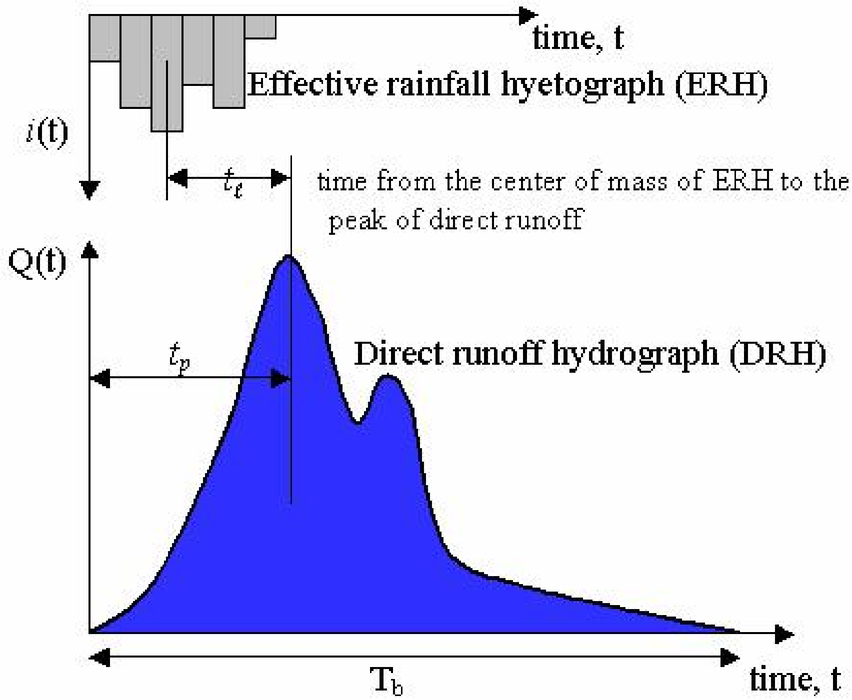
UNIT HYDROGRAPH FROM COMPLEX FLOOD HYDROGRAPH

Flood hydrographs with a single and sharp peak resulting from an intense and uniform rainfall are very uncommon. Often times, observed hydrographs contain multiple peaks of various magnitudes resulting from several spells of rainfall. UH, in such cases, are derived by

- Collins' Method
- Matrix method
- Instantaneous Unit Hydrograph



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UNIT HYDROGRAPH DETERMINATION BY COLLIN'S METHOD

This method uses trial and error approximations to compute UH from complex hydrograph. The basic steps involved in this method can be best gathered by an illustrative example. This example is available in excel file of this week schedule, which can be downloaded by participants.

Example

The direct runoff hydrograph at site-S and the effective hydrograph due to a particular storm over the catchment of river-R are tabulated below. We will use the given DRH and ERH to derive unit hydrograph because of 1mm effective rainfall. Catchment area is 8570 km^2.

Table 1: Ordinates of DRH

Time (Hrs)	Ordinates of DRH (cumec)
0	0
3	35
6	240
9	675
12	1140
15	1375
18	1345
21	1145
24	890
27	645
30	445
33	290
36	190
39	110
42	70
45	33
48	10
51	0

Table 2: Ordinates of ERH

Time (Hrs)	Ordinates of ERH (mm)
0-3	2.4
3-6	5.6
6-9	3.0
Rainfall Excess	11.0

Note- The effective rainfall hyetograph blocks are for 3 hour intervals. Therefore, the unit duration of the unit hydrograph thus derived will be of 3-hour unit duration.

The following steps guide the reader to obtain UH by this method.

i) From the observed flood hydrograph and observed rainfall hyetograph DRH and ERH are separated as explained earlier;

ii) The ordinates of the DRH at different time are written under Column 2 of the Table 3;

iii) Trial & error of determining UH ordinates begins with selection of first set of trial values representing ordinates of UH. This we can do by dividing the ordinates of DRH with 11mm effective rainfall, and recording them under column 3 of the Table 3;

iv) The ordinates of the assumed unit hydrograph are summed up as 785.4 cumec. But the sum of ordinates of the unit hydrograph at 3 hour interval, ΣU for a catchment area of 8570 km^2 should be:

Catchment Area * rainfall depth (1mm) = volume of UH

= Sum of UH ordinates, ΣU * time interval between UH ordinates

Hence,

$$\sum U = \frac{A*1mm}{(t\ hr *60*60)} = (8570*10^6 \text{ (in } m^2) * 10^{-3} \text{(in m)})/(3*60*60 \text{ sec}) = 793$$

where A = area of catchment in km^2, =8570 km^2, & t = time interval in hour = 3 hours

v) In order to satisfy condition stated above, all the assumed ordinates are multiplied by a factor 1.01 (793/785.4) and the adjusted values are entered in col.4 of the table;

vi) The ordinates on adjusted UH (col.4) are multiplied by 2.4, first burst of rainfall, and are written under col.5. Similarly, the ordinates of adjusted unit hydrograph (col.4) are multiplied by 3.0 mm, third burst of rainfall, and are reflected in col.7 after shifting it by 6 hours. Why this shift is warranted here? - It is so because 3.0 mm rainfall begins after 6hrs from the start of the storm.;

vii) The ordinates in col.5 and 7 are now added together and written under column 8. This gives the DRH resulting from rainfall excess (<u>2.4mm + 3.0mm</u>) excepting the largest one, i.e. <u>5.6mm</u>;

viii) The DRH ordinates obtained in col.8 are deducted from the ordinates of DRH in col.2, and are noted down in col.6. This is a DRH as a result of 5.6mm of rainfall. First ordinate due to 5.6 mm rainfall is made zero as this represents the beginning of contribution of 5.6mm rainfall;

ix) The values in col.6 are divided by 5.6mm to give the unit hydrograph, and are fed in col.9. Since, this is a UH, it is necessary to re-validate its volume against condition stipulated under Para (iv) above. The sum of ordinates of this UH as written in col. 9 is now 777. These ordinates are, therefore, multiplied by 1.02 (793/777) to readjust its values as reflected in col.10.

x) The weighted average of the two unit hydrographs (the assumed one as in column 4 and calculated one as in col.10) is reached in the following manner:

$$\text{Weighted average of the ordinates of the UH} = \frac{\text{Ordinates of adjusted assumed hydrograph} \times \text{Sum of Rainfall excess excepting largest one} + \text{Ordinates of calculated UH} \times \text{Largest Effective RF}}{\text{Total Rainfall Excess}}$$

For example, first ordinate will be,

$$\frac{0x5.4 + 0x5.6}{(5.4+5.6)} = 0$$

the 2nd ordinate will be,

$$\frac{3.0 \times 5.4 + 34.0x5.6}{(5.4 + 5.6)} = 19.0$$

xi) The weighted average ordinates are written under column 11 of the Table-3. The ordinates of the unit hydrograph in column 11 and column 4 are now compared for differences. If significant differences are noticed between the two, 2nd iteration begins with unit hydrograph ordinates of column 11; and this will occupy column 4. Thus, the process is iterative and will go on till the differences in the assumed and calculated unit hydrographs reduce to insignificant level.

Table 3: Calculation Table

		DERIVATION BY COLLINS METHOD					Runoff in cumecs			
CATCHMENT AREA		km² 8570		EFFECTIVE RAINFALL		0 TO 3	3TO 6	6 TO 9	TOTAL	
(DRH in cumec)				in mm		2.4	5.6	3	11	
1	2	3	4	5	6	7	8	9	10	11
TIME	DRH	UH1	Corrected UH	DUE TO 2.4	due to 5.6	DUE TO 3.0	(2.4+3.0)	cal UH	UH2	WEIGHTED AV=UH3
0	0	0	0	0			0			
3	35	3	3	8	0		8	0	0	0
6	240	22	22	53	187	0	53	33	34	19
9	675	61	62	149	516	10	159	92	94	59
12	1140	104	105	251	823	66	317	147	150	107
15	1375	125	126	303	886	186	489	158	161	134
18	1345	122	124	297	734	314	611	131	134	130
21	1145	104	105	252	514	379	631	92	94	108
0	890	81	82	196	323	371	567	58	59	82
3	645	59	59	142	187	316	458	33	34	58
6	445	41	41	98	102	245	343	18	19	38
9	290	26	27	64	48	178	242	9	9	25
12	190	17	17	42	25	123	165	5	5	15
15	110	10	10	24	6	80	104	1	1	9
18	70	6	6	16	2	52	68	0	0	5
21	33	3	3	7	0	30	38	0	0	3
0	10	1	1	2	0	19	22	0	0	1
3	0	0	0	0	0	9	9	0	0	0
					0	3	3	0	0	0
					0	0	0	0	0	0
		785.4	793.5					777	794	794
Ordinates of UH		793.5								

The ordinates of the derived unit hydrograph are tabulated below. At times, it is possible that the lower part of the unit hydrograph may not be uniform. In such cases, it is smoothened and redrawn smoothly with least variations in a manner that its volume should fulfil condition of Para (iv).

Table 4: Ordinates of UG

Ordinates of UG	
Time (hrs)	Ordinates (Cumecs)
0	0
3	17
6	63
9	114
12	134
15	127
18	105
21	80
0	57
3	39
6	25
9	17
12	9
15	1
18	0
sum	**794**

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UNIT HYDROGRAPH DERIVATION BY DE-CONVOLUTION OF DIRECT RUNOFF HYDROGRAPH

The discrete convolution equation allows the computation of direct runoff, Qn given excess rainfall, P_m and the unit hydrograph, U_{n-m+1}.

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

The reverse process, called de-convolution, can be utilised to derive a unit hydrograph given data on P_m and Q_n. Suppose that there are 'M' pulses or burst of rainfall excess and 'n' pulse of direct runoff in the storm considered; then N equation can be written for Q_n, n = 1,2,….,n, in terms of (n - m + 1) unknown values of the unit hydrograph.
If Q_n and P_m are given and U_{n-m+1} is required, the set equations is over determined, because there are more equations (N) than unknowns (n - m + 1). The term n ≤ M in the equation restrains the total nos. of P*U terms for Qn. In first case, when n is ≤ M, m = 1, 2, ...n; while in case n is > M, m = 1,2, ….M.

(**<u>For more details, reader may refer to Applied Hydrology by Ven Te Chow)</u>**

Let us derive term for Q_1, assuming total number of rainfall pulse, M = 3

Here, n =1 ≤ (M =3), hence, m =1, therefore,

$$Q_1 = P_1 . U_{1-1+1} = P_1 . U_1$$

For Q_5, n = 5 > (M =3), hence, m = 1, 2, & 3, therefore,

$$Q_5 = P_1 . U_{5-1+1} + P_2 . U_{5-2+1} + P_3 . U_{5-3+1}$$

$$= P_1 . U_5 + P_2 . U_4 + P_3 . U_3$$

Following this concept, a set of all ordinates of DRH can be derived as below.

Q_1 =	**P_1U_1**								
Q_2 =	P_2U_1 +	P_1U_2							
Q_3 =	P_3U_1 +	P_1U_2 +	P_1U_3						
……..									
Q_M =	P_MU_1 +	$P_{M-1}U_2$ +	+ ……+	P_1U_M					
Q_{M+1} =	0 +	P_MU_2	+ ……+	P_2U_M +	P_1U_{M+1}				
…….									
Q_{N-1} =	0 +	0 +	+ ……+	0 +	0 +	+ ……+	P_MU_{N-M+1}	$P_{M-1}U_{N-M+1}$	
Q_N =	0 +	0 +	+ ……+	0 +	0 +	+ ……+	0	P_MU_{N-M+1}	

This can also be arranged in matrix form with an equation

$$[Q]^{n*1} = [P]^{n*(n-m+1)} . [U]^{(n-m+1)*1}$$

Q_1 =	**P_1**	**0**	**0**	**0**	**0**	**0**	**0**		**U_1**
Q_2 =	**P_2**	**P_1**	0	0	0	0	0		U_2
Q_3 =	**P_3**	**P_2**	**P_1**	0	0	0	0		U_3
Q_4=	0	**P_3**	**P_2**	**P_1**	0	0	0	*	U_4
Q5 =	0	0	**P_3**	**P_2**	**P_1**	0	0		U_5
--	--	--	--	--	--	--	--		
--	--	--	--	--	--	--	--		U_{n-m+1}
Q_n=	0	0	0	0	0	0	**P_m**		

With known values of Q and P, matrix U is determined by following equation

$$[U] = ([P]^T[P])^{-1} [P]^T [Q]$$

Where, $[P]^T$ is transpose matrix of P, and $([P]^T[P])^{-1}$ indicates inverse matrix of $[P]^T . [P]$. To bring out the meaning/strength of this method, let us attempt to derive UH ordinates through an example.

Example

An observed hydrograph with rainfall excess is given as under. The time interval is 3 hours between readings. Catchment area is 7092 km^2.

Table 1: Ordinates of DRH and Effective Rainfall Hyetograph (ERH)

Time	Rainfall Excess (mm)	Hour	Observed Discharge (m^3/s)
0-3	0.5	1	5
3-6	1.2	2	35
6-9	0.9	3	90
		4	203
		5	816
		6	1602
		7	1138
		8	302
		9	275
		10	158
		11	65
		12	47
		13	12

Let us first define number of equations.

There are 3 pulses of rainfall so M = 3. There are 13 pulses of observed direct runoff so n = 13. The number of unit hydrograph ordinates is therefore, n - m +1 = 13 - 3 + 1 = 11 ordinates.

Applying this piece of information, set-up of matrices for Q, P and U appear as below

$$
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \end{bmatrix}
=
\begin{bmatrix}
P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_2 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_3 & P_2 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & P_3 & P_2 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & P_3 & P_2 & P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & P_3 & P_2 & P_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & P_3 & P_2 & P_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & P_3 & P_2 & P_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & P_3 & P_2 & P_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_3 & P_2 & P_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_3 & P_2 & P_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_3 & P_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_3
\end{bmatrix}
*
\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \\ U_{10} \\ U_{11} \\ \\ \end{bmatrix}
$$

Where, [Q] is 13 by 1 matrix with discharge ordinates; [P] is a 13 by 11 matrix having three rainfall impulse of given duration; and [U] is unknown matrix of 11 by 1 size whose ordinates are to be determined. In matrix form, information of tabular chart reduces to

$$[Q]_{13*1} = [P]_{13*11} \cdot [U]_{11*1}$$

To solve this problem for [U] matrix, we choose only 11 equations to obtain 11 unknown UH ordinates using following equation.

$$[U] = [P]-1\ [Q]$$

Result obtained using MS excel shown next has generated a few negative terms in the falling limb of UH; and this has to be adjusted by the reader in such a way as to total volume of UH must be equal to the volume of runoff emerging form the catchment as a result of 1mm uniform &

effective rainfall over it. This example, thus, underlines the likelihood of a few negative and abnormal terms in the calculation and need subsequent readjustment.

U_1	10
U_2	46
U_3	52
U_4	199
U_5	1061
U_6	300
U_7	-352
U_8	900
U_9	-1000
U_{10}	1078
U_{11}	-657

Adjusted unit hydrograph is shown below.

Time Interval (1 hr)	Unit Hydrograph (m^3/s)
1	10
2	46
3	51.6
4	199.4
5	1060.7
6	299.4
7	128.2
8	117.4
9	37.5
10	14.7
11	5.2

At this stage, reader is advised to refer to the excel sheet for familiarizing themselves with calculation part of this example.

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UNIT HYDROGRAPH BY CLARK MODEL

The Clark model uses two parameters, time of concentration, 'Tc', and storage constant, 'K', and a time-area histogram concept. Before, we set out for UH by Clark model, let us first familiarize ourselves with these new terminologies.

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DETERMINATION OF TIME CONCENTRATION, Tc

The first parameter, time of concentration is the time taken by a water particle from the hydraulically farthest point to the basin outlet. An estimate of this travel time is the time from the end of runoff producing rainfall over the basin to the inflection point on the recession limb of the direct Runoff Hydrograph (DRH). Because of complexities involved in rainfall-runoff process, all such incidences never produce reproducible 'Tc' nor can we ever know true Tc. And therefore, an averaged value of 'Tc' from observed data should be considered for analysis. Alternatively, any empirical equation, most valid for area under study is recommended for use. Some of the empirical equations normally used for estimation of this parameter are as under:

Methods	Equation	Parameters
Kirpich	$T_c = m0.000323L^{0.77}S^{-0.385}$	Here, Tc is in hr, Longest flow path, L in meter & equivalent slope, S is dimensionless. Secondly, coefficient, m for overland flow is = 1.0 on bare earth = 2.0 on grassy earth = 0.4 on asphalt
Kerby	$T_c = \left[\frac{6.56LN}{3\sqrt{S}}\right]^{1/2.14}$	N: Roughness constant. Here, Tc is in min, L in m & S is dimensionless
California	$T_c = \left[\frac{0.8702L^3}{H}\right]^{0.385}$	H: Difference in elevation of the river in m, Tc is in hr & L in km

(For more discussion on 'Tc', reader may please refer to Hydrologic Analysis and Design by Richard H. McCuen and Technical Reference Manual of HEC-HMS software).

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DETERMINATION OF BASIN STORAGE COEFFICIENT (K)

This coefficient represents the temporary storage of precipitation excess in the watershed as it drains to the outlet point. K is expressed in terms of time. If observed short interval flood discharges at site are available, then hydraulically, K is estimated by using the formula:

$$K = \frac{Q_1}{(Q_2 - Q_1)/t}$$

Where;

Q_1:	correspond to the discharge, after separating base flow, at the point of inflection on the recession limb of flood hydrograph.
Q_2:	correspond to the discharge, after separating base flow, after time t on the recession limb of flood hydrograph.
t:	time interval between Q_1 and Q_2.

Adopting a suitable base flow, the value of K can be computed for different hydrographs and an average value can be worked out.

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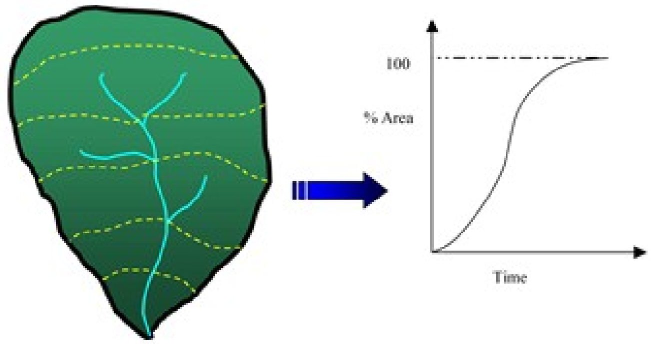
TIME-AREA CONCEPT

The time-area diagram represents the areas that will contribute to the flow at the outlet over successive periods of time. Once the time of concentration is known, lines of equal time interval called *isochrones* can be drawn with assumption that time of travel is directly proportional to distance from the outlet to isochrones. In the picture shown here, dashed lines in yellow mark isochrones.

The US Army Corps of Engineers recommends following formula to develop time area table/diagram for estimation of inflow from areas bounded between successive isochrones. An example at the end of this discussion explains the use of this equation.

$$\frac{A_t}{A} = \begin{cases} 1.414\left(\dfrac{t}{t_c}\right)^{1.5} & for. t \le \dfrac{t_c}{2} \\ 1 - 1.414\left(1 - \dfrac{t}{t_c}\right)^{1.5} & for. t \ge \dfrac{t_c}{2} \end{cases}$$



The time-area diagram is considered as the inflow to a hypothetical reservoir (S = KO; this implies that there is absence of wedge storage) and routed through the reservoir to obtain the outflow hydrograph which is the required instantaneous UH for the basin. Before routing, inflow from incremental areas between isochrones is converted into discharge units by following equation;

for 1mm rainfall, $\quad \mathbf{I_i} = \dfrac{0.278ai}{t}$

Where 'ai' is the area in km2 and 't' is the routing period in hours.

Here, we will take a pause to understand as to how this model produces an outflow with time-area concept; and inflow is generated by each time-area zone due to instantaneous 1mm effective rainfall. Let us consider the uppermost part of the catchment. Being uppermost part of the watershed, it does not receive any outflow (O_0=0). Instead, it produces I1 runoff which takes 1 hr (if tc for the catchment is 6hrs; and isochrones are separated by 1hr each) to reach at the tip of area just below it with a magnitude of O_i routed by following equation.

$$\mathbf{O_i = CI_i + (1\text{-}C)O_i\ \text{-}1}$$

Where **C = (t/(K+0.5t)), here, t is routing interval in hr, k is basin storage coefficient;** and I_i and O_i are the inflow and outflow at the end of period t_i.

In other words, for uppermost area, outflow is obtained by

$$\mathbf{O_1 = CI_1 + (1\text{-}C)^*\ 0}$$

Now, let us consider the area downstream of first one. Like above, this part generates inflow, I_2 and receives O_1 as outflow from upper area. Using routing equation, this area will produce O_2 as

$$O_2 = CI_2 + (1\text{-}C)^*\ O_1$$

This process is continued till we reach terminal point of the catchment/study area.

The IUH can be converted to a unit hydrograph of same unit duration as routing interval simply averaging two instantaneous hydrographs lagged by the selected duration that is,

$$Q_i = 0.5(O_i + O_{i-1})$$

To obtain unit hydrograph for durations other than routing interval (provided that it is exact multiple of routing interval t. The following equation is used.

$$Q_i = \frac{1}{n}(0.5O_{i-n} + O_{i-n}+1 + ...+ O_{i-1} + 0.5O_i)$$

Where Q_i is ordinate of unit hydrograph of desired duration D = nt.

The computation of a 1-hr unit hydrograph by this method is described ahead with an example.

Example

Derive UH for a project site using Clark model with physiographic characteristics of the basin given in Table 1. No observed flood hydrograph is available at the project site.

Table 1: Physiographic Parameters

Parameter	Definition	Formula	Unit	Value
L	Length of longest main stream along the river course	Measured on Topographical Map	km	52.80
L_c	Length of longest main stream from a point opposite to centroid of the catchment area to intake site	Measured on Topographical Map	km	27.40
A	Rain fed Area	Based on satellite imageries and below 4600 m elevation measured on Topographical Map	km²	810
S	Equivalent Stream Slope	$S = \frac{\sum L_i(D_{i-1} + D_i)}{L^2}$	m/km	24.90

Solution

Determination of The time of concentration (T_c)

Since, no observed flood hydrograph is available at project dam site, Tc, time of concentration is determined using the Kirpich, the Kerby and the California formulae as shown in Table 2. The average value as 6.0 hour based on 3 formulae is adopted for project catchment up to project site.

Table 2: Determination of the Time of Concentration (T_c)

Method	Equation	River upto project site
Kirpich	$T_c = 0.01947 L^{0.77} S^{-0.385}$	5.8 hr
Kerby	$T_c = \left[\frac{6.56LN}{3\sqrt{S}}\right]^{1/2.14}$ N = 0.65	7.5 hr
California	$T_c = \left[\frac{0.8702L^3}{H}\right]^{0.385}$ *H =2790m*	4.4 hr
	Average tc	**6.0 hr**

Since the time of concentration adopted is 6 hr, the catchment area has been divided into six isochrones representing 1 hr equal travel time. The equation developed by U.S. Army Corps of Engineers is used to estimate the time-area relationship of the watersheds. Table 3 presents the resulting Time-Area relationship for river upto dam site.

Table 3: Time-Area Relationship for project catchment upto project site

Isochrones	Travel Time (hr)	Cumulative Area (km^2)	Incremental Area (km^2)
1	1	78	78
2	2	220	142
3	3	405	185
4	4	590	185
5	5	732	109
6	6	810	78

Note- For HEC-HMS software savvy readers, calculation reflected in table 3 is not essential. HEC-HMS software needs two key parameters, i.e. Tc and K to be input appropriately in its environment, and rest of calculation and UH output are handled by software in no time.

From the Table 3, the value of catchment area can be used developing inflow for each computational time interval using excel sheet.

Determination of Basin storage coefficient (K)

Since the site specific observed short interval discharge data is not available for dam site, therefore basin storage coefficient is estimated based on **regional values of K** available for concerned river.

For estimating the value of storage coefficient 'K', observed complex flood hydrographs from 3rd to 7th September 2001, 22nd to 25th Jul-2002, September 2004 flood hydrographs at one G&D site on the same river have been used. The detail is given below:

Table 4: Detail of Storage coefficient (K), G&D site

Flood Hydrograph Period	Flood Discharge at Point of Inflection after deducting base flow (Q_1) (m^3/s)	Flood Discharge less base flow (Q_2) (m^3/s)	(t_2-t_1)	K (Hr)	Base flow (m^3/s)
3-7th Sept, 2001	1376	1216	1	8.6	750
22/07/2002	2452	2263	1	13.0	750
25/07/ 2002	1660	1300	1	4.6	750
07/2004	743	674	1	10.8	650
			Average	9.2	

Therefore from Table 4, the Storage coefficient K can be taken as 9.0.

Assuming a base flow of 450 m^3/s, the value of K has been computed for different hydrographs at another site on the same river as shown in Table 5. The average value of 8.0 hr has been adopted for observed hydrographs at this G&D site, an average value of attenuation constant K equal to 8.0 hr has been worked out based on 4 flood events.

Table 5: Detail of Storage coefficient (K), second G&D site, same River

Flood Hydrograph Period	Flood Discharge at Point of Inflection after base flow (Q_1) (m^3/s)	Flood Discharge after base flow (Q_2) (m^3/s)	(t_2-t_1)	K (Hr)	Base flow (m^3/s)
27/08/2006	330	285	1	7.3	450
09/09/2006	170	147	1	7.4	450
11/07/2007	342	304	1	9.0	450
20/07/2007	391	343	1	8.1	450
			Average	8.0	

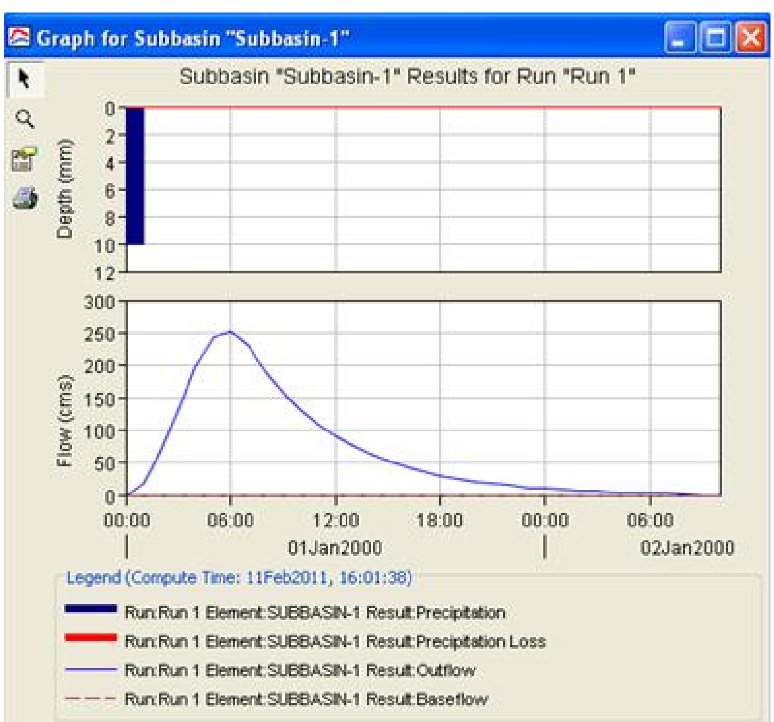
The K is estimated based on the fact that for a given regional value of this parameter at a particular site, the dimensionless parameter $\frac{K}{K+T_c}$ **(also called as attenuation ratio)** approximately remains constant. The value of this parameter for second and first G&D sites is given in the Table 6:

Table 6: Regional detail of K & T_C, river catchment

Site	K	T_C	$\frac{K}{K+T_c}$
Second G&D site	8	8	0.50
First G&D site	9	10	0.474

Above value of dimensionless parameter $\frac{K}{K+T_c}$, which is almost equal, suggests in favour of conclusion given by U.S. Army Corps of Engineers that the value of this dimensionless parameter remains more or less constant over a region. Based on this fact, the value of K estimated for given project site is **5.4 hr (for Tc = 6.0hr).**

Having determined catchment area, time of concentration, Tc and storage coefficient, K, a model depicting basin is developed in HEC-HMS and basin is subjected to 10mm=1cm effective rainfall for 1-hr duration. Resulting hydrograph, i.e. 1-hr duration UH and its ordinates are available below:



Project: UNIT TEST
Simulation Run: Run 1 Subbasin: Subbasin-1

Start of Run: 01Jan2000, 00:00 Basin Model: Basin 1
End of Run: 02Jan2000, 10:00 Meteorologic Model: Met 1
Compute Time: 11Feb2011, 16:01:38 Control Specifications: Control 1

Date	Time	Precip (MM)	Loss (MM)	Excess (MM)	Direc... (M3/S)	Base... (M3/S)	Total... (M3/S)
01Jan2000	00:00				0.0	0.0	0.0
01Jan2000	01:00	10.00	0.00	10.00	18.4	0.0	18.4
01Jan2000	02:00	0.00	0.00	0.00	67.5	0.0	67.5
01Jan2000	03:00	0.00	0.00	0.00	133.4	0.0	133.4
01Jan2000	04:00	0.00	0.00	0.00	198.1	0.0	198.1
01Jan2000	05:00	0.00	0.00	0.00	241.9	0.0	241.9
01Jan2000	06:00	0.00	0.00	0.00	253.0	0.0	253.0
01Jan2000	07:00	0.00	0.00	0.00	228.6	0.0	228.6
01Jan2000	08:00	0.00	0.00	0.00	189.8	0.0	189.8
01Jan2000	09:00	0.00	0.00	0.00	157.7	0.0	157.7
01Jan2000	10:00	0.00	0.00	0.00	130.9	0.0	130.9
01Jan2000	11:00	0.00	0.00	0.00	108.7	0.0	108.7
01Jan2000	12:00	0.00	0.00	0.00	90.3	0.0	90.3
01Jan2000	13:00	0.00	0.00	0.00	75.0	0.0	75.0
01Jan2000	14:00	0.00	0.00	0.00	62.3	0.0	62.3
01Jan2000	15:00	0.00	0.00	0.00	51.7	0.0	51.7
01Jan2000	16:00	0.00	0.00	0.00	43.0	0.0	43.0
01Jan2000	17:00	0.00	0.00	0.00	35.7	0.0	35.7
01Jan2000	18:00	0.00	0.00	0.00	29.6	0.0	29.6
01Jan2000	19:00	0.00	0.00	0.00	24.6	0.0	24.6
01Jan2000	20:00	0.00	0.00	0.00	20.4	0.0	20.4
01Jan2000	21:00	0.00	0.00	0.00	17.0	0.0	17.0
01Jan2000	22:00	0.00	0.00	0.00	14.1	0.0	14.1
01Jan2000	23:00	0.00	0.00	0.00	11.7	0.0	11.7
02Jan2000	00:00	0.00	0.00	0.00	9.7	0.0	9.7
02Jan2000	01:00	0.00	0.00	0.00	8.1	0.0	8.1
02Jan2000	02:00	0.00	0.00	0.00	6.7	0.0	6.7
02Jan2000	03:00	0.00	0.00	0.00	5.6	0.0	5.6
02Jan2000	04:00	0.00	0.00	0.00	4.6	0.0	4.6
02Jan2000	05:00	0.00	0.00	0.00	3.8	0.0	3.8
02Jan2000	06:00	0.00	0.00	0.00	3.2	0.0	3.2
02Jan2000	07:00	0.00	0.00	0.00	2.7	0.0	2.7
02Jan2000	08:00	0.00	0.00	0.00	2.2	0.0	2.2

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INSTANTANEOUS UNIT HYDROGRAPH

The instantaneous Unit Hydrograph is defined as unit hydrograph or infinitesimally small duration. In other words, IUH is the direct runoff hydrograph at the outlet of the catchment resulting from 1 unit (1mm) of rainfall falling over the catchment in zero time. Of course, this is only a fictitious situation and a concept to be used in hydrograph analysis.

Derivation of IUH

There are various methods for the determination of an IUH from the given effective rainfall hyetograph and direct runoff hydrograph. But the most common is the model suggested by Nash in 1957. Nash proposed a conceptual model by considering a drainage basin as 'n' identical linear reservoirs in series. By routing a unit inflow through the reservoirs a mathematical equation for IUH can be derived.

The ordinate of the IUH at time t is given by,

$$U(t) = \frac{1}{K*(n-1)!} *(t\backslash k)^{n-1} .e^{-t\backslash k}$$

Where,

n = no. Of the reservoir; and

K= a reservoir constant, also called as storage coefficient.

The values of K and n in Nash model can be evaluated by the method of moments by using the following relations:

$$M_{DRH1} - M_{ERH1} = nK$$

$$M_{DRH2} - M_{ERH2} = n(n+1)\,K^2 + 2\,nK\,M_{ERH1}$$

Where,
M_{DRH1}= First moment arm of Direct Runoff Hydrograph (DRH)
M_{ERH1}= First moment arm of Effective Rainfall Hyetograph (ERH)
M_{DRH2}= Second moment arm of DRH
M_{ERH2}= Second moment arm of ERH

The unit of the ordinates of IUH is per sec (sec-1). When the ordinates are multiplied by the total volume of runoff (in cubic meters) resulting from 1mm of rainfall over the catchment area, the unit will be cumecs.

Derivation of Unit Hydrograph from IUH

For finding the unit hydrograph from IUH, the area under the IUH is plotted with respect to time at the point. The entire area from the start of IUH at different time interval gives points of S Curve. If a unit hydrograph of T hour duration is required, the S Curve so arrived at is shifted by T hour and the difference in the ordinates of the two S Curves is computed and divided by T. The resulting curve forms the unit hydrograph of T hour duration. To illustrate derivation of UH by Nash method, an example is presented ahead.

Example

A storm of mild intensity was experienced in the catchment of river Baitarani during the period from 26.9.75 to 28.9.75. The rainfall was rather non-uniform. The average hourly rainfall over the catchment and the resulting observed discharge at Anandpur site are furnished in Table 1 & 2. The area of catchment of River Baitarani up to Anandpur is 8570 sqkm.

With this set of data pertaining to this storm, an IUH followed by the unit hydrograph of one-hour unit duration have been computed below:

Table1

Time	Total Rainfall in mm
1	2
2	4.65
3	5.34
4	0.1
5	0.1
6	4.62

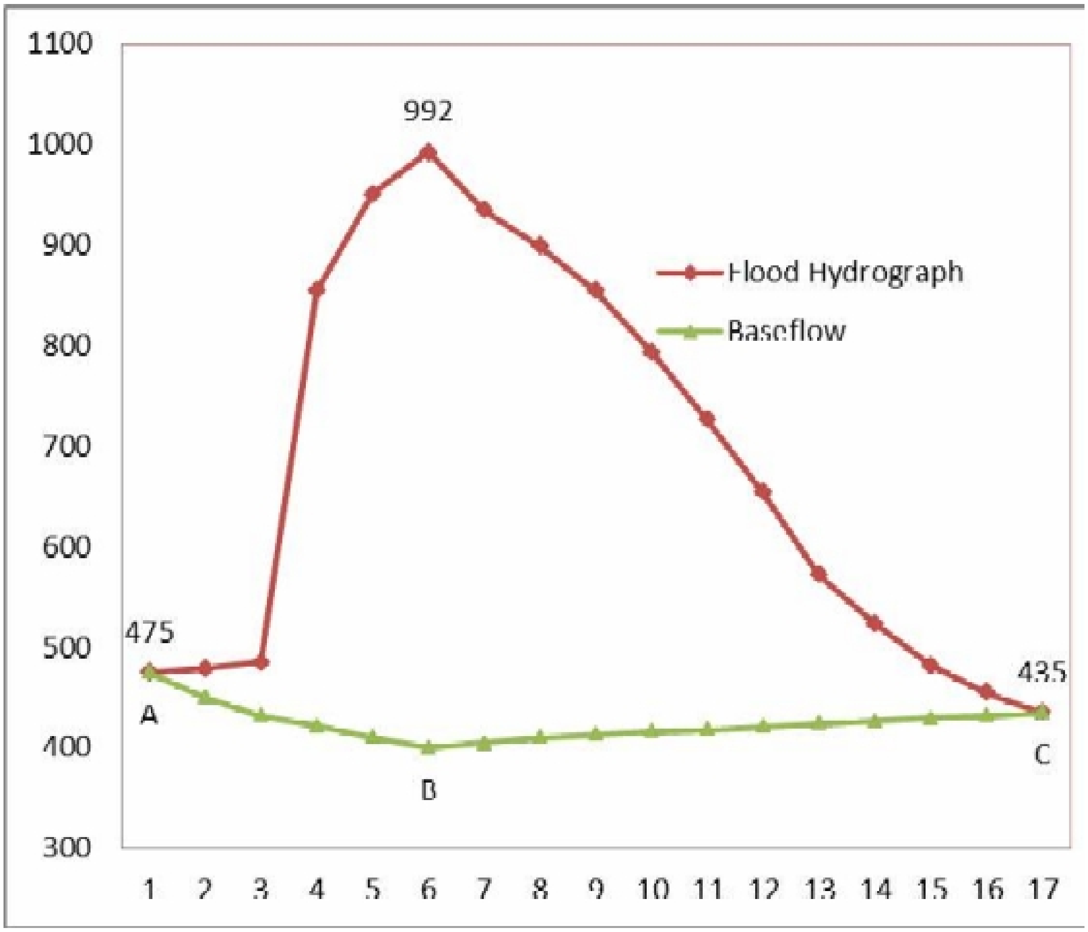
Table-2

Date	Time in hour	*Ordinates of Flood* Hydrograph (cumec)
26.9.75	0	420
	3	500
	6	495
	9	490
	12	475
	15	479
	18	485
	21	854
27.9.75	0	950
	3	992
	6	934
	9	899
	12	854
	15	793
	18	726
	21	654
28.9.75	0	572
	3	523
	6	482
	9	455
	12	435

The various steps involved in the procedure are as follows:

1. Separation of base flow to find DRH

- The anticipated recession curve of the smaller peak just before this flood was continued till the time of the peak, i.e. point B.
- A suitable point was chosen on the falling limb at a distance of about 2t to 2.5t where 't' is the time from the rise of flood hydrograph to the peak.
- These points B & C were joined by a straight line. The curve thus separates the base flow from total flood.
- The base flow thus separated is deducted from the corresponding ordinates of the flood hydrograph to get the direct runoff hydrograph (DRH).



2. Separation of Rainfall Excess from Total Rainfall

The total hourly rainfall (average over the basin) is shown in the table above. Since the storm under consideration is in the month of September and there was a heavy storm in August and yet another storm of smaller intensity in early September, the loss may be considered to take place at a uniform rate.

Let the loss be at the uniform rate of 'X' mm/hour.

Additionally, the total volume of of DRH is worked out to be

Volume of DRH = 4344 cumecs* 3hrs

Where, 4344 is total sum of DRH ordinates and 3hrs is time interval between two adjacent DRH ordinates.

$$\textbf{Total direct runoff} = \frac{13032 \times 3.6}{8570} = 5.475\text{mm},$$

Where, 8570 is the area of catchment of River Baitarani up to Anandpur in sqkm.

Now let the loss rate, X be more than 2 mm/hour.

Time	Total Rainfall	Loss	Effective Rainfall
1.	2	2	0
2.	4.65	X	4.65-X= ***1.605***
3.	5.34	X	5.34-X= **2.295**
4.	0.1	0.1	0
5.	0.1	0.1	0
6.	4.62	X	4.62-X= ***1.575***
	14.61	Total	14.61-3X

Equating **14.61-3X=5.475**

We get, **X = 3.045 mm**

Substituting the value of X, the effective rainfall at different time is obtained.

For the purpose of analysis, the hourly ordinate will involve a lot of computational work, and therefore, only the three hourly rainfall ordinates have been considered. The hourly rainfalls are assumed to be uniform during three hours and accordingly, the ordinates of DRH and ERH are shown in Table A.1 below

Table A.1

Date	Tim	Ordinates of Flood		
		Hydro	Base	DRH
26.9.7	0	420	420	0
	3	500	500	0
	6	495	495	0
	9	490	490	0
	12	475	475	0
	15	479	450	29
	18	485	432	53
	21	854	422	432
27.9.7	0	950	410	540
	3	992	400	592
	6	934	405	529
	9	899	410	489
	12	854	413	441
	15	793	416	377
	18	726	418	308
	21	654	421	233
28.9.7	0	572	424	148
	3	523	426	97
	6	482	429	53
	9	455	432	23
	12	435	435	0

	Ordinates of ERH
0-3	1.3
3-6	0.525

3. Calculation of n and K

M_{ERH1} = first moment arm of ERH

$$= \frac{\text{first moment arm of ERH about O}}{\text{Area of ERH}}$$

$$= \frac{1.3x3x[3/2]+0.525x3x[3+3/2]}{1.3x3+0.525x3}$$

$$= \frac{5.85+7.088}{3.9+1.575} = \frac{12.938}{5.475} = 2.363$$

M_{ERH2} = Second moment arm of ERH

$$= \frac{\text{Second moment of ERH about O}}{\text{Area of ERH}}$$

$$= \frac{1.3 \times 3 \times [3/2]^2 + 0.525 \times 3 \times [3+3/2]^2}{5.475}$$

$$= \frac{8.775 + 31.894}{5.475} = \mathbf{7.428}$$

M_{DRH1} = First moment arm of DRH

$$= \frac{\text{First moment arm of DRH about O}}{\text{Area of DRH}}$$

$$= \frac{[29x3x3] + [54x3x6] + [432x3x9] + [540x3x12] + [592x3x15] + [529x3x18] + [489x3x21] + [432x3x24] + [377x3x27] + [308x3x30] + [235x3x33] + [154x3x36] + [97x3x39] + [53x3x42] + [23x3x45]}{[29x3] + [54x3] + [432x3] + [540x3] + [592x3] + [529x3] + [489x3] + [432x3] + [377x3] + [308x3] + [235x3] + [154x3] + [97x3] + [53x3] + [23x3]}$$

$$= \frac{268740}{13032} = \mathbf{20.622}$$

M_{DRH2} = 2^{nd} moment arm of DRH

$$= \frac{2^{nd}\text{ moment arm of DRH about O}}{\text{Area of DRH}}$$

$$= \frac{[29x3x3^2]+[54x3x6^2] + [432x3x9^2] + [540x3x12^2] + [592x3x15^2] + [529x3x18^2] + [489x3x21^2] + [432x3x24^2] + [377x3x27^2] + [308x3x30^2] + [235x3x33^2] + [154x3x36^2] + [97x3x39^2] + [53x3x42^2] + [23x3x45^2]}{13032}$$

$$= \frac{6537510}{13032} = \mathbf{501.651}$$

Now, $\mathbf{nK}$ = $M_{DRH1} - M_{ERH1}$

= 20.622 – 2.363 = 18.259

and $n(n+1)K^2$ = $M_{DRH2} - M_{ERH2} - 2nK\, M_{ERH1}$

= 501.651 – 7.428 – 2X18.259 X 2.363

= 407.931

From the above two equations, we get **K = 4.083 hr & n = 4.471**

Hence, for analysis the values of K and n may be taken as 4 hr and 4 respectively *(In case of very sandy soil characteristics of the catchment 'n' may be taken as 5 if its value works out to be 4.47, whereas for hilly or semi-hilly regions of catchment, n should be 4).*

4. To estimate the ordinates of IUH

Once n and K are found out, the ordinates of the IUH can be found very easily by using the relation

$$U(t) = \frac{1}{K*(n-1)!}(t/K)^{n-1} \cdot e^{-t/K}$$

This will give ordinates in units of Sec^{-1}. To find the ordinates in cumec, it is multiplied by catchment area contribution due to 1mm effective rainfall. Thus for first time unit U(1),

Area = 8570 * 10^6 m^2, K = 4 * 60 * 60 sec, (n-1)! = (4-1)! = 6, t = 1 hr. *Pl note that in term (t/K), K would be 4hr as we are considering t in hr.* This step is repeated for rest of time units to generate IUH. The ordinates of IUH are given in Table below:

Time (hr)	Computed ordinates of IUH (cumec)	Smoothened ordinates of IUH (cumec)	IUH lagged by 1hr	Ordinates of 1-hr UH (cumec) obtained by col3+col4/2
1	2	3	4	5
0	0.00	0		0
1	1.21	1	0	1
2	7.52	8	1	4
3	19.76	20	8	14
4	36.48	36	20	28
5	55.49	55	36	46
6	74.68	75	55	65
7	92.36	92	75	84
8	107.37	107	92	100
9	119.06	119	107	113
10	127.19	127	119	123
11	131.84	132	127	130
12	133.30	133	132	133
13	132.00	132	133	133
14	128.39	128	132	130
15	122.99	123	128	126
16	116.24	116	123	120
17	108.59	109	116	113
18	100.39	100	109	105
19	91.95	92	100	96
20	83.52	84	92	88
21	75.30	75	84	80
22	67.43	67	75	71

23	60.00	60	67	64
24	53.09	53	60	57
25	46.74	47	53	50
26	40.94	41	47	44
27	35.71	36	41	39
28	31.02	31	36	34
29	26.84	27	31	29
30	23.14	23	27	25
31	19.88	20	23	22
32	17.03	17	20	19
33	14.55	15	17	16
34	12.39	12	15	14
35	10.53	10	12	11
36	8.92	9	10	10
37	7.54	8	9	9
38	6.36	6	8	7
39	5.36	5	6	6
40	4.50	4	5	5
41	3.78	4	4	4
42	3.16	3	4	3
43	2.64	3	3	3
44	2.20	2	3	2
45	1.84	2	2	2
46	1.53	2	2	2
47	1.27	2	2	2
48	1.05	1	2	1
49	0.87	1	1	1
50	0.72	0	1	0
			0	

It will be seen that the last ordinate is never zero. This is because of the fact that the recession of IUH is generally defined by an exponential function which has zero value only at infinity. Hence, the recession is terminated at a suitable point and volume adjusted

5. Derivation of 1 Hr. duration Unit Hydrograph from IUH

The t^{th} hour ordinates of the 1-hour duration unit hydrograph can be very easily found by simply taking the average of t^{th} hour and $(t-1)^{th}$ hour ordinates of the IUH. Column 2 of the Table above gives the ordinates of the IUH (rounded off figures) the column 5 gives the ordinates of the 1-hour duration unit hydrograph.

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UNIT HYDROGRAPH FOR UNGUAGED CATCHMENTS

More often than not, project sites suffer from inadequate length of hydro-meteorological data or even no data to deduce any reliable hydrological inputs or to develop reliable transfer function in the form of a UH. In several cases, constraint of this kind is not uncommon in India, which compels an engineer to resort to develop a synthetic unit hydrograph. A Synthetic Unit Hydrograph (SUH) takes its shape and size according to the physical characteristics of the basin under study. There are few methods of developing SUH. Some of the common methods for derivation of synthetic unit hydrograph for a basin are as follows:

- ***Snyder method***

Among several known methods for development of synthetic unit hydrograph, the one suggested by F. F. Snyder (1938) is most commonly used. Snyder analyzed a large number of hydrographs from drainage basins in the Appalachian Mountain region in the United States, ranging in area from 25 sq. km. to 25,000 sq. km.

To sketch a unit hydrograph, it is necessary to know the time of the peak, the peak flow and the time base. The elements must be determined for every particular or regional location of the drainage basin. Snyder proposed the following empirical formula for the lag time (Hr.) from mid-point of effective rainfall duration t_r to peak of a unit hydrograph:

$$t_p = C_t (L. L_c)^{0.3}$$

in which t_p = the basin lag in hours, from midpoint of effective rainfall duration t_r to peak of a unit graph:

L= the length of the main stream from the outlet to the divide in kms;

L_c =the distance from the outlet to a point on the stream nearest to the centroid of the basin; and C_t = a coefficient

The location of the center of area may be determined by cutting the basin outline from cardboard and marking the point of intersection of plumb lines drawn with the map suspended from different corners. The coefficient C_t varies from 1.0 to 2.2 with lower values associated with basins of steeper slopes.

For the standard duration of effective rainfall t_r , Snyder proposed:

$$t_r = \frac{t_p}{5.5}$$

For the rains of this duration, he found that synthetic unit hydrograph peak Q_p in cumecs may be obtained from the equation:

$$Q_p = \frac{C_p}{t_p} A$$

And in cumecs/sq km by the relation

$$Q_p = \frac{C_p}{t_p}$$

Where, A = the drainage area in square kms.

C_p = coefficient ranging from 4.0 to 5.0

Q_p = peak flood in cumec.

For the time base T (in days) of the synthetic unit hydrograph U.S. Army Corps of Engineer adopted the following expression

$$T = 3 + 3 . \frac{t_p}{24}$$

These equations are sufficient to construct a synthetic unit hydrograph for a storm of duration t_r

The value of Snyder's coefficients C_t and C_p are found to vary considerably depending upon the topography, geology and climate. Snyder indicated that the coefficient C_t is affected by basin slopes S. Linsely, Kohler and Paulhus have suggested and expression for t_p in which the basin slope S has been considered.

$$t_p = C_t \left[\frac{LL_c}{\sqrt{S}}\right]^N$$

Where, N=0.38 and C =1.2 for mountainous drainage areas; 0.72 for foothills; and 0.35 for valley areas.

- *From gauged to ungauged basin by transposition of unit hydrograph*
    - If unit hydrographs are available for several areas adjacent to a basin for which a unit hydrograph is required but for which necessary data are lacking, then transposition of

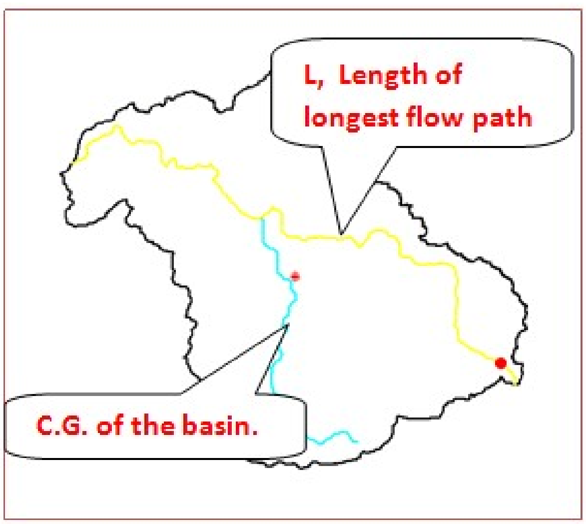
available unit hydrograph will ordinarily give better results than resorting to a synthetic procedure. Sherman originally proposed that the ordinates and abscissas of unit hydrograph for similar basins might be assumed to be proportional to the square roots respective drainage areas. Further details are available in any textbook on applied hydrology.

- *Based on set of equations recommended by Flood Estimation Reports (FER)*

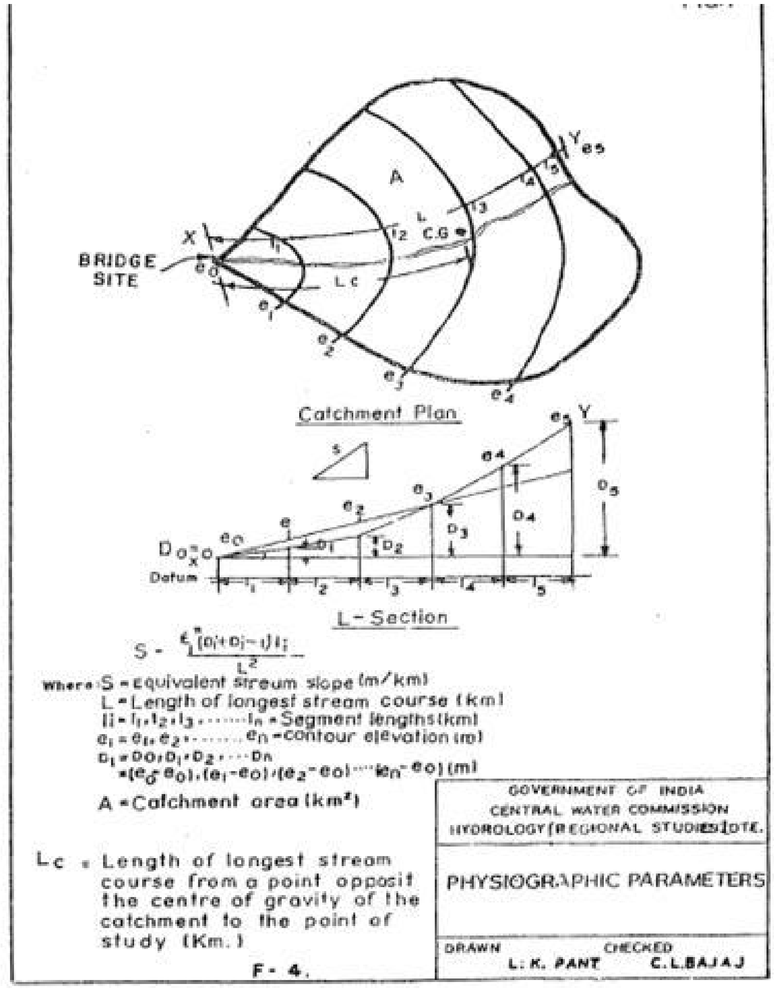
FER for 26 hydro-meteorologically homogenous subzones in India are reports jointly brought by Central Water Commission (CWC); India Meteorological Department (IMD); Research, Design & Standard Organization (RDSO); and Ministry of Shipping & Transport (MoST). In each report, a number of mathematical relationships between physiographic parameters and components of unit hydrograph, derived on multiple regression technique, exist. The step-by-step procedures as to how to develop UH based on FER are illustrated in paragraphs to follow.

Step-1 Physiographic parameters

1. Location of catchment area to be identified from Survey of India topo-sheet and measure the catchment area (A)
2. Measure the length of the longest stream in Km. (L)
3. Length of the longest stream from a point opposite to C.G. of catchment to the point of study in Km. (L_c)
4. Compute Equivalent Slope in m/Km. (S_{eq})



To determine equivalent slope, reader may look at following plot which displays longitudinal profile and formula used for the purpose. We will use this formula a little later to calculate Seq.



With advances in information technology in recent years and also with the availability of ***Di****gital* ***E****levation* ***M****odel (DEM), distillation of physiographic parameters is relatively faster and accurate. Illustrated example in later part of this chapter demonstrates the application of GIS technique to deduce these parameters. The DEM data are available for free at sites*

http://srtm.csi.cgiar.org/SELECTION/inputCoord.asp

http://www.gdem.aster.ersdac.or.jp/search.jsp

http://glcf.umd.edu/data/landsat/

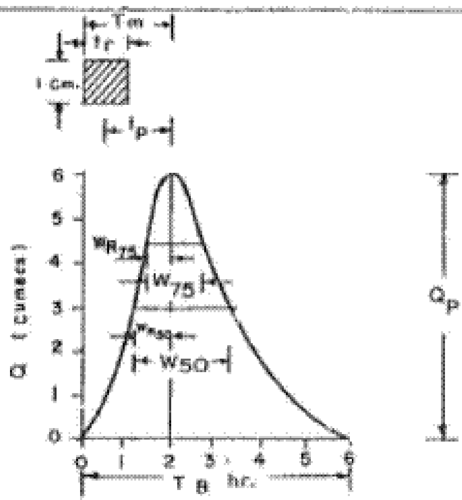
http://glcfapp.glcf.umd.edu:8080/esdi/index.jsp

Step-2 To construct 1-hour SUH

Availability of physical parameters of catchment enables one to estimate the components of SUH using following SUH equations for a Zone, say X;

Qp in cumec =	**$0.905(A)^{0.758}$ Where, A is basin area in km²**
Qp in cumec/km² =	QP/A
W50 in hr =	$2.304(q_p)^{-1.035}$
W75 in hr =	$1.339\ (q_p)^{-0.978}$
WR50 in hr =	$0.814(q_p)^{-1.018}$
WR75 in hr =	$0.494\ (q_p)^{-0.966}$
Tp in hr =	**$2.87(q_p)^{-0.839}$**
Tm in hr =	**$t_p + (t_r/2)$**
TB in hr =	**$2.447\ (t_p)^{1.157}$**

FIG. - 2.



U. G = Unit Graph

t_r = Unit Rainfall Duration adopted in a specific study (hr.)

T_m = Time from the start of rise to the peak of the U.G (hr.)

Q_p = Peak Discharge of Unit Hydrograph (Cumecs.)

t_p = Time from the Centre of Effective Rainfall duration to the U.G Peak (hr.)

W_{50} = Width of the U.G measured at the 50% of peak discharge ordinate (hr.)

W_{75} = Width of the U.G measured at 75% of peak discharge ordinate (hr.)

W_{R50} = Width of the rising limb of U.G measured at 50% of peak discharge ordinate (hr.)

W_{R75} = Width of the rising limb of U.G measured at 75% of peak discharge ordinate (hr.)

T_B = Base width of Unit Hydrograph (hr.)

A = Catchment Area (Sq.km.)

q_p = Q_p/A = Cumec per sq.km.

Notations/components used in above table are used to construct an SUH using a plot shown here. If the volume of UH so defined deviates from the volume of runoff generated by catchment because of 1cm rainfall excess, falling limb of the hydrograph is suitably modified without altering the points of synthetic parameters such that the volume of UH equals the theoretical value. The SUH ordinates at one hour interval after corrections are taken as the final estimate of SUH. Succeeding example presents stepwise procedure to develop SUH of 1-hr duration.

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ILLUSTRATIVE EXAMPLE

Important Note

1. **This example is based on hypothetical equations and is used to demonstrate the steps required to design a synthetic UH. For real case studies, users are requested to refer to Flood Estimation Report relevant to the area under study. Such reports are available with CWC and concerned State Govt. departments.)**
2. **Readers ignorant of application of GIS software and Geo-HMS add-on package will find it difficult to follow initial steps of this example.**

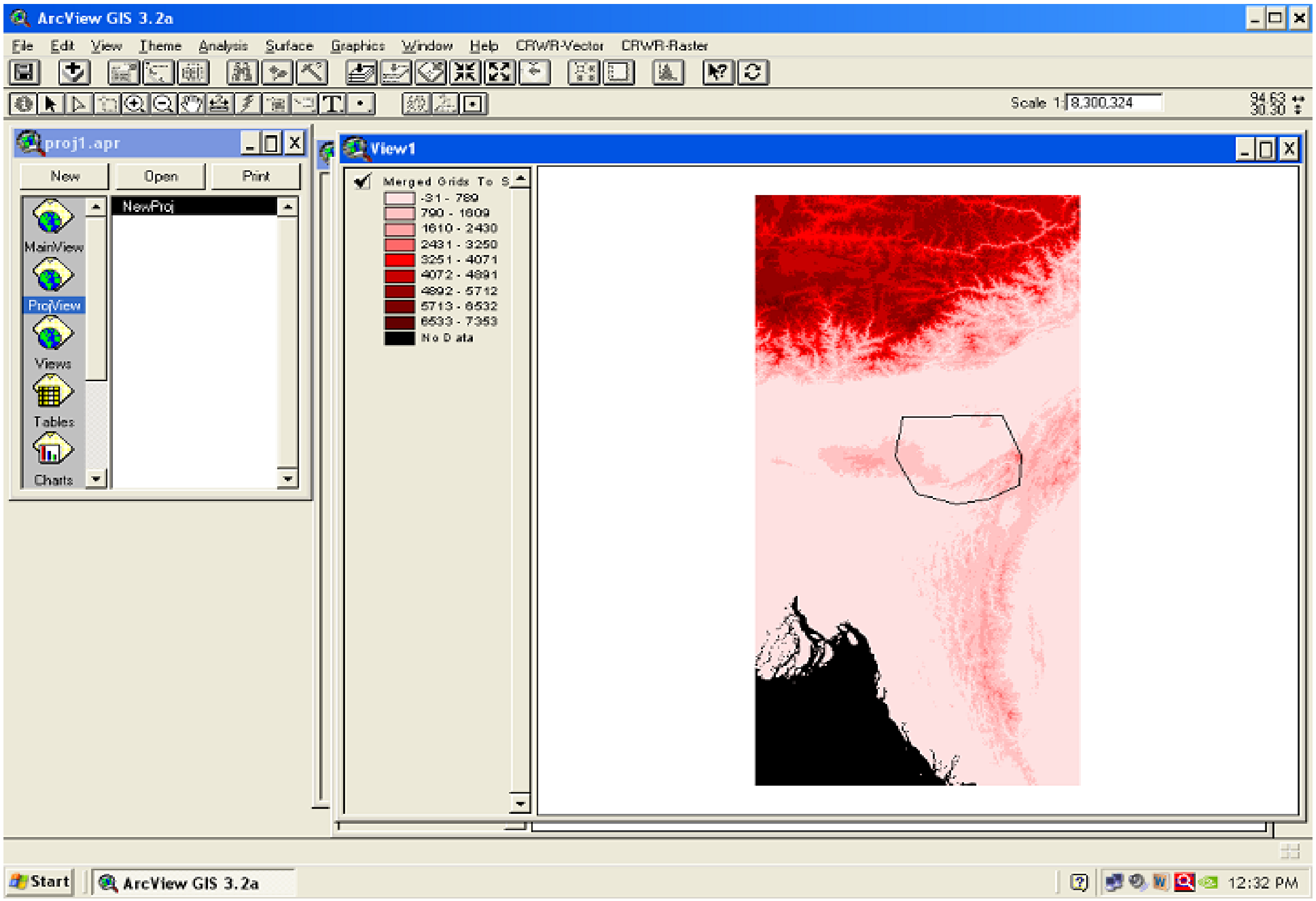
The particulars of a catchment/project site are as follows:

(i)	Name of watershed	:	A
(ii)	Name of Tributary	:	X
(iii)	Location	:	Lat $25^0\ 47'00''$ Long $93^0\ 04'48''$
(vi)	Topography	:	Moderate Slope

Step-1: Physiographic Parameters

The Survey of India released topo-sheets marked with contours is primary requisites to extract basin physical parameters. Unquestionably, this process is tedious and fraught with possible errors. Alternatively, Windows based GIS presentation Fig.1 software, such as Arc View 3.X or similar packages can be used to extract these parameters through analysis of DEM. To showcase strength of GIS based analysis of basin, two SRTM grids 55_7 and 55_8 are first merged followed by extraction of a part from whole DEM, which is likely to cover outlet point and its contributing area.

This relatively small sized patch of DEM was imported in Geo-HMS window (an extension package that works on Arc View GIS 3.2a) for terrain and hydrological analysis. A series of options on Geo-HMS supported menus lead users to successfully obtain a network of streams and corresponding catchment areas. This process is termed as terrain processing.



Having reached a stage as shown in Fig.2, it is possible in the system to locate project by defining outlet at pre-determined latitude and longitude. In Fig.2, hypothetical project location is displayed by a point in red. A hydrologist is concerned about area shedding water at this outlet; and therefore, only this area is abstracted at this stage for hydrological analysis subsequently.

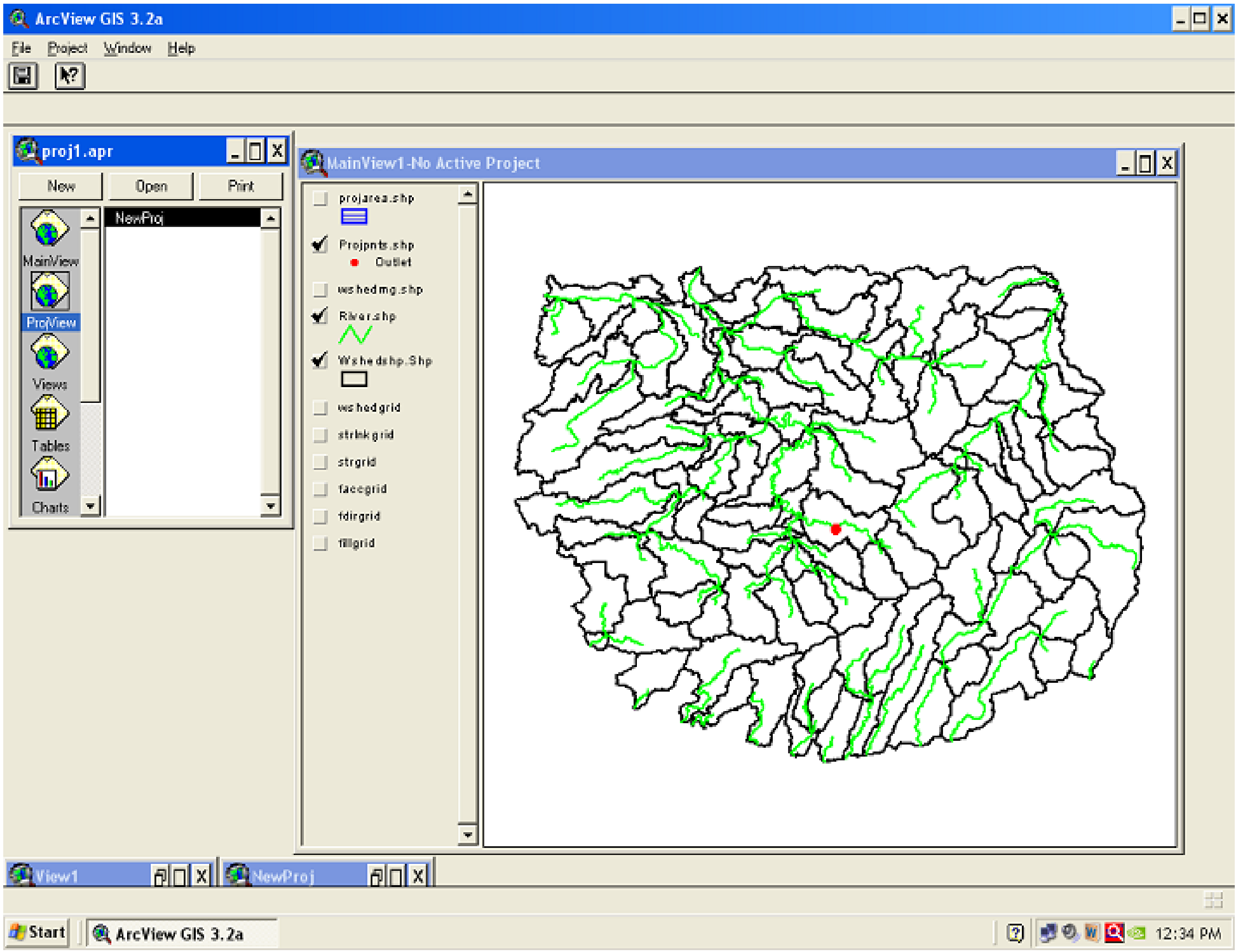


Fig.3 displays the output of hydrological processing containing delineated watershed for project location; the location of centre of gravity (CG) of the basin; longest flow path. These features of this basin can be exported to HEC-HMS software for developing design flood hydrograph or any other hydrological analysis. A set of information extracted for estimating SUH equations are as below:

Area	-	606.52 km^2
Length of longest flow path, L	-	56.4 km
Centroidal flow path, Lc	-	28.2 km

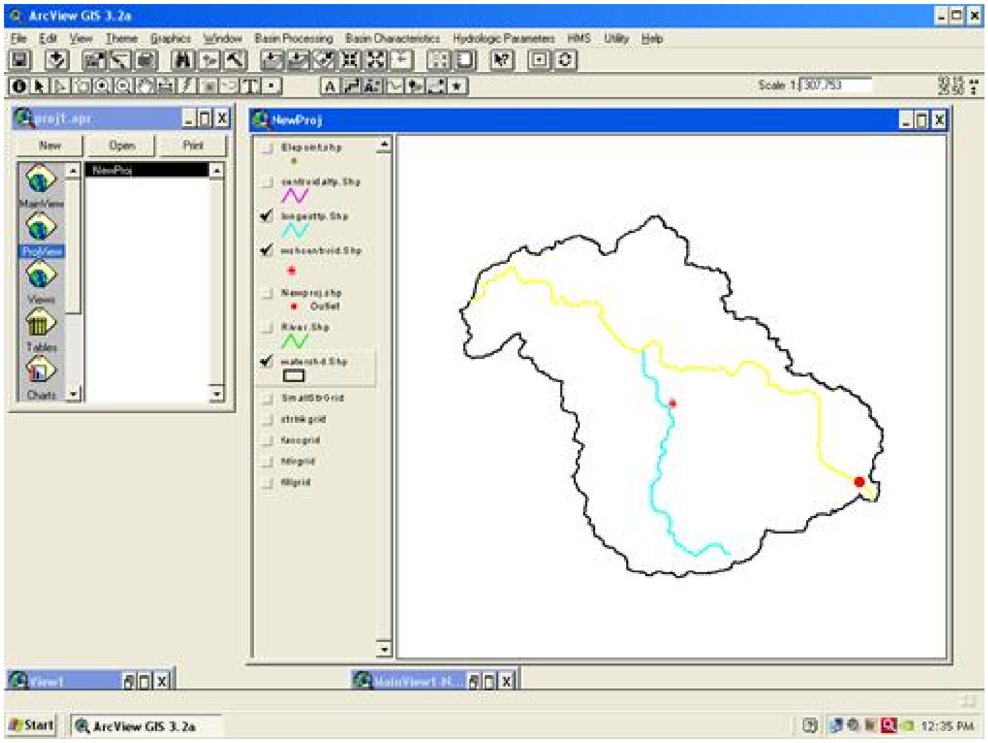


Fig.4 plots the longitudinal profile of the river/basin along longest flow path. This plot and table presented below are used to estimate equivalent slope.

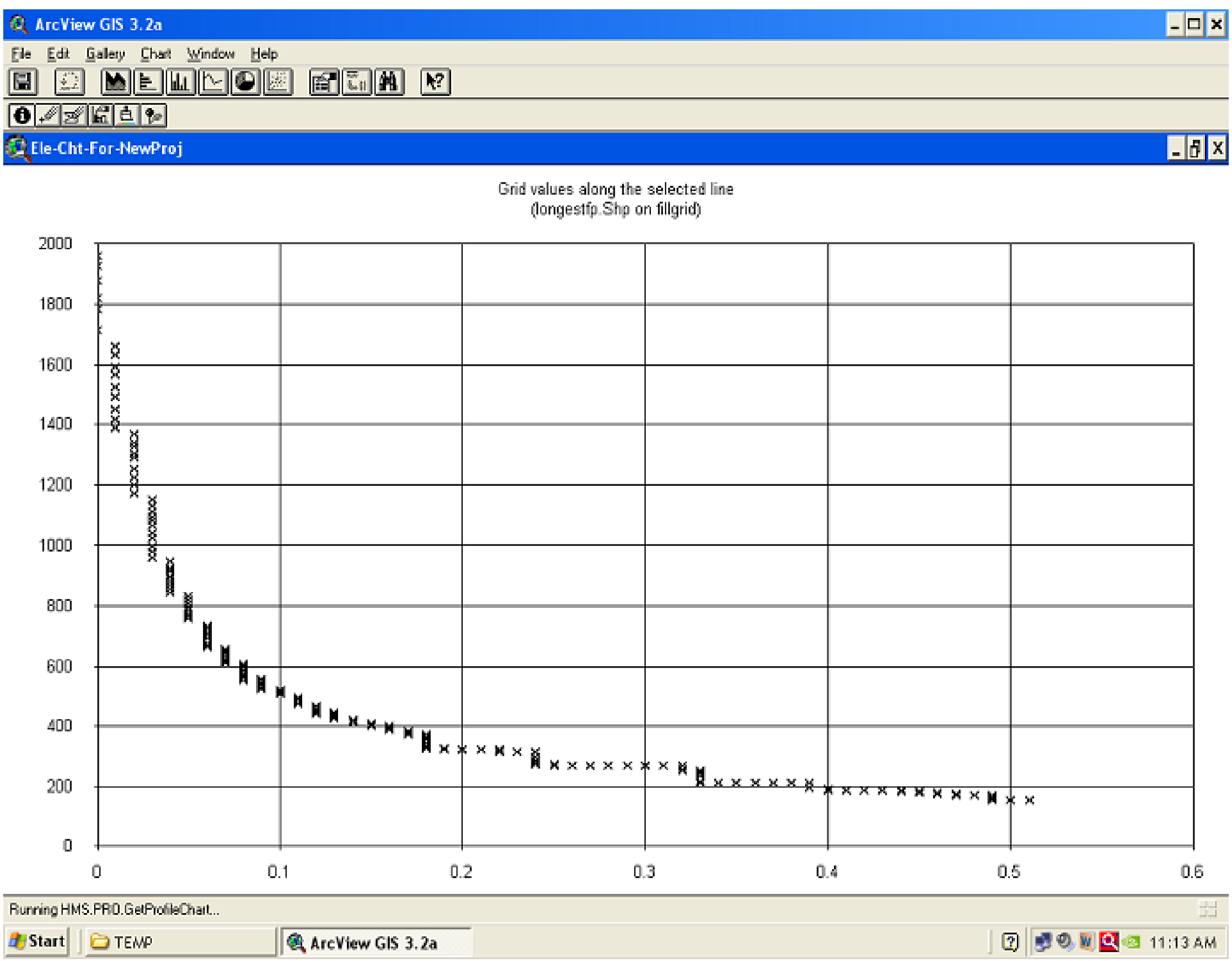


Table-Computation of Equivalent slope

(Refer to L-section of river in Fig.4)

Sl No.	Reduced Distance RD (km) (from project site)	Reduced Level RL (m)	Length of each segment(L_i) Km	Height above datum (D_i)(m)=176m	Di+Di-1*	Li(Di+Di-1)
1	0.0	176		0		
2	16.7	189	16.69	13	13	216.92
3	27.8	272	11.12	96	109	1212.54
4	38.9	325	11.12	149	245	2725.44
5	50.1	509	11.12	333	482	5361.89
6	56.4	1959	6.34	1783	2116	13417.19
			L=56.40 km		$\Sigma li^*(D_i+D_{i-1})$=	22934

Datum (RL of river bed at point of study)= 176 m

$$S_{eq} = \Sigma L_i(D_i+D_{i-1}) \;/L^2 = 22934/(56.40)^2$$

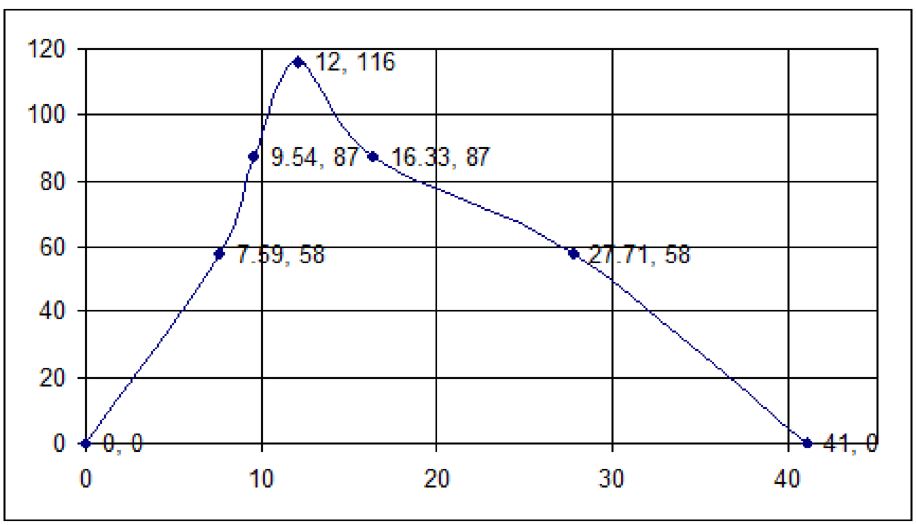
$$=7.21 \text{ m/km}$$

Step-2 : 1 hr Synthetic UH parameters generated by 1cm effective rainfall

SUH parameters as given below are computed by using equations given in step 2 are as following:

Q_p	=	$0.905\ (A)^{0.758}$	=	$0.905\ (606)^{0.758}$	=	116.42 say 116.0 cumec
q_p	=	(Q_p/A)	=	$(116.0/606.52)$	=	0.19 cumec/km.^{2}
t_p	=	$2.87\ (q_p)^{-0.839}$	=	$2.87\ (0.19)^{-0.839}$	=	11.56 say 11.50 hrs.
W_{50}	=	$2.304\ (q_p)^{-1.035}$	=	$2.304\ (0.19)^{-1.035}$	=	20.12 hrs
W_{75}	=	$1.339\ (q_p)^{-0.978}$	=	$1.339\ (0.19)^{-0.978}$	=	6.79 hrs.
WR_{50}	=	$0.814\ (q_p)^{-1.018}$	=	$0.814\ (0.19)^{-1.018}$	=	4.41 hrs
WR_{75}	=	$0.494 (q_p)^{-0.966}$	=	$0.494\ (0.19)^{-0.966}$	=	2.46 hrs
T_B	=	$2.447\ (t_p)^{1.157}$	=	$2.447\ (11.50)^{1.157}$	=	41.29 hrs say 41.00 hrs
T_m	=	$t_p + (t_r/2)$	=	$11.50 + (1/2)$	=	12.00 hrs.

An SUH based on the estimated parameters in step -2 is shown below. The discharge ordinates of this graph at 1 hr interval are multiplied by 1 hr and are summed up to ascertain whether it agrees with principle of UH. Raw UH so arrived overestimates the runoff; and therefore its falling limb ordinates are readjusted in a manner to represent basin's true UH.



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CHANGING UNIT HYDROGRAPH DURATIONS

The unit duration of the UH derived from various records may not be alike. In order to compare and average them, it is necessary to convert all of them to the same unit duration.

There may be two types of cases;

1. When UH of shorter unit duration't' is known and a UG of longer duration T is to be derived where T is a multiple of t, i.e., T=nt such that n = 1, 2, 3, ...n. This can be achieved simply by the principle of super-imposition. Let us grasp it with an example. We have an 1 unit UH of 2-hr duration; and it needs to be converted to a 4-hr duration. Please refer to the table shown here. The 2 hour UH is displaced by two hours; and is added to first UH. The summed up UH is a result of 2 unit rainfall spread over (2hr+2hr) = 4hr duration. To obtain 1 unit 4hr duration of UH, Last column is divided by 2.

Time (hr)	Q	Displaced UHG	Sum	4 hour UHG
0	0		0	0
1	2		2	1
2	4	0	4	2
3	6	2	8	4
4	10	4	14	7
5	6	6	12	6
6	4	10	14	7
7	3	6	9	4.5
8	2	4	6	3
9	1	3	4	2
10	0	2	2	1
11		1	1	0.5
12		0	0	0

2. Otherwise, the UH of other duration can be derived by S-curve.

   The S-curve is a hydrograph produced by a continuous effective rainfall at a constant rate for an indefinite period. The S-hydrograph can be constructed by summing up a series of identical UHs spaced at intervals equal to the unit duration of the UH. After the S-hydrograph is constructed, the UH of a given duration is derived with following procedures.

   Assume that the S-hydrograph derived is due to effective rainfall intensity of $1/t_0$ mm/hour. Now, advance or offset the position of S-hydrographs for a period equal to the desired duration of UH, say t_n hours; and tabulate the difference between ordinates of original S-hydrograph and offset S-hydrograph. This will be the hydrograph due to $(1/t_0) * t_n = t_n/t_0$ mm of rainfall occurring in tn hours. Divide ordinates of the hydrograph thus obtained by t_n/t_0. The resulting hydrograph will be the UH of tn hour duration.

3. HEC-HMS software uses UH as transfer function to generate response of a basin following occurrences of rainfall. In HEC-HMS, UH of known duration, say t-hr is to be fed in the HEC-HMS keeping its ordinates spaced at t-hr apart. For example, if UH of 1mm rainfall derived is of 3hr duration, ordinates of UH must be entered at 3hr interval. Software automatically converts this UH to other duration according to rainfall interval chosen by user. Thus, conversion of a UH from one duration to another is omitted.

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AVERAGING UNIT HYDROGRAPH

Three distinct unit hydrographs for Anandpur site on river Baitarani, as tabulated below, have been developed by employing different techniques of UH derivation as highlighted in preceding paragraphs. The figures in bold and italic mark the peak value of respective columns. A cursory look at ordinates recorded under three methods reveals that the peaks, time base, and time of occurrence of three unit hydrographs differs from one another. As a matter of fact, if various storms are considered for development of unit graph for the same catchment, a marked variation will be observed especially in the peak as well as the time of occurrence of the peak. Therefore, it had better derive an average unit hydrograph for practical use. If several unit hydrographs are averaged by averaging concurrent ordinates, it is highly probable that the resulting average unit graph has a broader, and a quite possibly a lower peak than any of the individual graphs.

Time in hrs	Single Storm Method	Clarke Method	IUH Method	Averaged UH ordinates
0	0	0	0	0
3	8	15	6.3	8
6	32	60	46.3	34
9	122	118	99	103
12	144	134	128.3	130
15	156	120	132	141
18	110	105	112.2	119
21	70	80	87.7	83
24	52	50	64	59
27	34	37	44	40
30	26	24	29	28
33	18	16	19	20
36	12	9	11	14
39	6	5	7	8
42	3	2	4	4
45	0	0	2.3	2
48	0	0	1.7	0
51	0	0	0.3	
54	0	0	0	

The correct average unit hydrograph should be obtained by locating the average peak and the average time of occurrence of the peak and sketching a menu unit hydrograph having an area equal to 1mm of runoff and resembling the individual graph as much as possible.

In the backdrop of discussion above, pattern of averaged UH is determined as below. To find the average unit hydrograph, the average peak was found to be 141 cumecs and average time of occurrence of peak was estimated to be 15 hours. Similarly the average base length is estimated to be 48 hours.

$$\text{Peak of averaged UH} = \frac{(156 + 134 + 132)}{3} = 141 \text{ cumec}$$

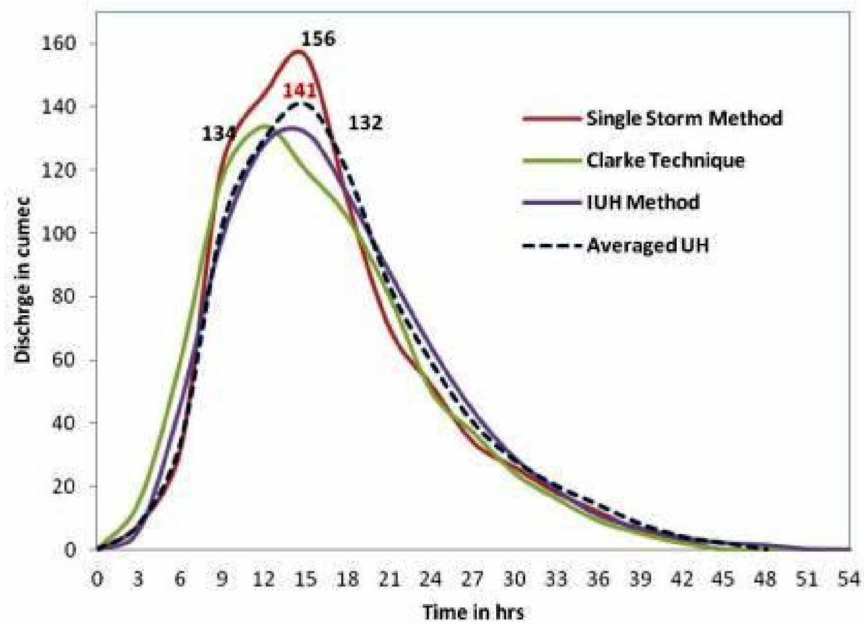
$$\text{Average time to peak} = \frac{(15 + 12 + 15)}{3} = 14 \text{ hrs}$$

$$\text{Average time base} = \frac{(42 + 42 + 51)}{3} = 48 \text{ hrs}$$

Now that essential component of UH are estimated, a suitable unit hydrograph is drawn in a manner such that:

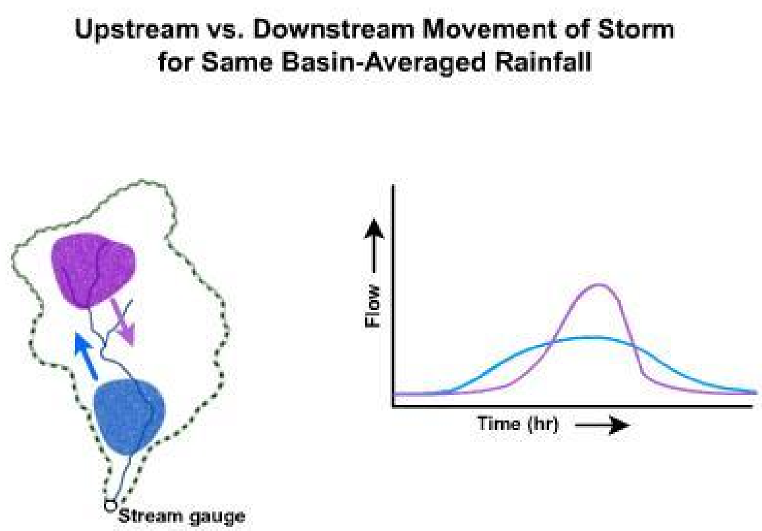
- The area of the unit hydrograph is equal to 1mm; and
- The shape resembles the shape of the three individual hydrographs.

The average unit hydrograph thus obtained is shown by dotted line.



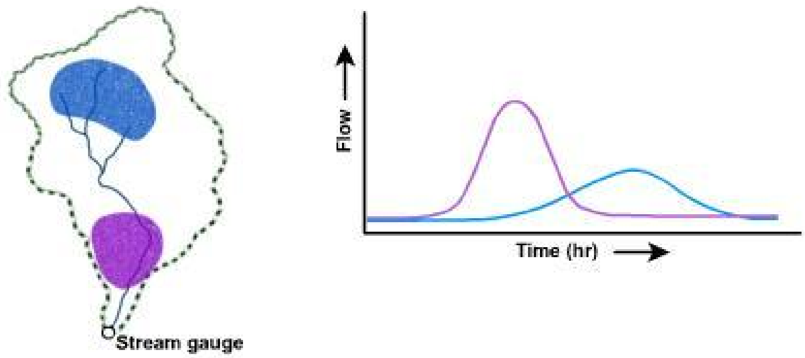
However, the averaging of the unit hydrograph can't be resorted to all cases. It has been observed that for practical purposes the shape of the unit hydrograph is hugely governed by factors, such as amount of effective rainfall, rainfall distribution pattern, and the storm movement etc

This aspect of UH is briefly demonstrated here with the help of a few diagrams. For more on this subject, readers are encouraged to refer to any good book on hydrology. Adjacent figure illustrates how the storm movement influences the shape of unit hydrograph. Yet another picture exhibits how the concentration of localized rainfall activity over the basin can significantly alter limbs and peak of UH. The third picture demonstrates the distinction between impact of a concentrated and heavy effective rainfall and uniform rainfall of same amount over the catchment.



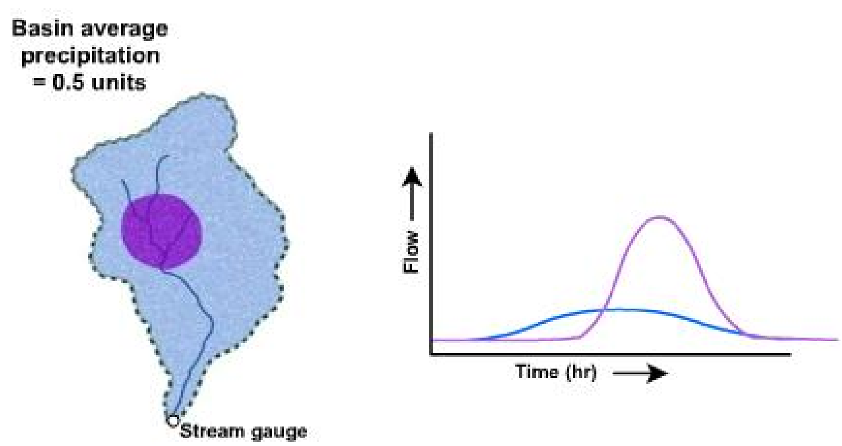
In brief, it is not necessary that similar features will be reflected in all the storms. As a matter of fact, the formation and distribution of the runoff is quite complex process in which large numbers of factors are involved.

Upstream vs. Downstream Rainfall Locations for Same Basin-Averaged Rainfall



Hence for the operational use, the scheme of the unit hydrograph is to be laid down after taking into account the primary influencing factors. It is not enough, and certainly not, to use one un-changeable unit hydrograph for formulation of flood forecast. Different unit hydrographs should be identified for the various conditions which have various influences on formation and time distribution of the runoff. These unit hydrographs may then be judiciously applied under different conditions.

Uniform Coverage vs. Intense Bull's-Eye for Same Basin-Averaged Rainfall



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